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Measuring Nonlinear Dependence Between Time Series
Based on Correlation Dimension

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(Received July 23, 2008)

Abstract: In this paper, it is proved that the correlation dimension estimate of a nonlinear dynamical system with its multivariate observation series is the same as that with its univariate observation series. Based on this result, an inference method is presented, and the Nonlinear Dependence Coefficient is defined. This method is designed for testing nonlinear dependence between time series, and can be used in economic analysis and forecasting. Numerical results show the method is effective.

Key words: chaos; nonlinear dynamics; correlation dimension; time series; nonlinear dependence; economic forecasting

1. Introduction

Complexity and fantasticality of the economic activity are induced generally by the interaction of different nonlinear relations within system. Some nonlinear methods obtain more and more attentions because of their ability to potentially explain the complexity economic behaviors. The discoveries of chaotic behavior and the research results of chaotic theory are especially outstanding. However, most research works in nonlinear economics, are with low dimensional nonlinear models to discuss the question of whether the bifurcate and chaos behaviors may emerge. There have been very fewer literatures to investigate the nonlinear explanation and inference methods with observation data. Here we consider the following discrete time nonlinear dynamical system \((h, F, x(0)):\)

\[
\begin{align*}
x(t) &= F(x(t - 1)), \quad x(0) \text{ given} \\
y(t) &= h(x(t)), \quad t = 1, 2, \ldots, n
\end{align*}
\]  

*This project is supported by the Key Research Project of Shanghai Municipal Education Commission (06ZZ34)
where
\[
F(x) = (F_1(x), \ldots, F_r(x))',
\]
\[
h(x) = (h_1(x), \ldots, h_p(x))',
\]
\[
F : \mathbb{R}^r \to \mathbb{R}^r, h : \mathbb{R}^r \to \mathbb{R}^p
\]
and
\[
x(t) = (x_1(t), \ldots, x_r(t))' \in \mathbb{R}^r
\]
\[
y(t) = (y_1(t), \ldots, y_p(t))' \in \mathbb{R}^p
\]
are state variable and observation variable respectively. Generally we have \(p=1\). There have been a lot of literatures to discuss nonlinear characteristics of the system when \(F(\cdot)\) is given. Unfortunately, we have generally no clue as to what form the \(F(\cdot)\) is. Sometimes we do not know what the relevant components of state vector \(x\) are. Even we can not sure the system dimension. What we have are some data of one observation variable of the dynamical system. However, univariate time series as a part of the multivariate system includes the information about all the variables of this economic system. We can investigate dynamical characteristics of the original economic system by this univariate time series with reconstruction techniques. A lot of literatures have discussed the method to estimate the system dimension by one-dimensional observation series \([1-3]\) of this system. Based on correlation integral, Brock, Dechert and Scheinkman presented a statistical inference method to test the nonlinear dependence within time series (one now refers to the BDS test). BDS test has been used in different ways, and especially have been used to test the predictability of time series in nonlinear economics \([4]\). This test can be used to test the predictability of the series in nonlinear economics. However, the idea to test nonlinear dependence between time series based on fractal dimension has not been considered before. Correlation dimension is one kind of fractal dimensions that is frequently used in applications. In this paper, it is proved that the correlation dimension estimate of a nonlinear dynamical system with its multivariate observation series is the same as that with its univariate observation series. Considering this result, the nonlinear dependence coefficient is defined to test the nonlinear dependence between time series. Numerical results show the method is effective.

2. Testing Nonlinear Dependence Based on Correlation Dimension

Consider system \((h,F,x(0))\). Let
\[
C_n(x, \varepsilon) = \frac{2}{n(n-1)} \cdot \sum_{1 \leq s < t \leq n} \theta(\varepsilon - ||x(t) - x(s)||)
\]
where
\[
\theta(a) = \begin{cases} 
0, & a < 0 \\
1, & a \geq 0
\end{cases}
\]
The correlation dimension \(D(x)\) of system \((h,F,x(0))\) is defined as
\[
D(x) = \lim_{\varepsilon \to 0} \lim_{n \to \infty} \ln C_n(x, \varepsilon) / \ln \varepsilon
\]
that
y
It can be proved that (See Theorem 1 and Corollary 1 of next section)

Here we suppose that

where

then

Especially we have

where

is generically used. In fact, (2) can be understood as the probability that two randomly and independently chosen points on the system orbit are within distance $\varepsilon$.

Suppose that \{y(t) : t = 1, \ldots, n\} are the observation data of $p$-dimensional observation vector $y$, let

where $m$ is called imbedding dimension, and correlation integral is defined as

where $N = n - m + 1$, so that the estimator of correlation dimension can be given by

It can be proved that (See Theorem 1 and Corollary 1 of next section)

Especially we have

Here we suppose that $p = 2$. If $y_1$ and $y_2$ come from the different dynamical systems (it means that $y_1$ and $y_2$ are independent), we have

where $P$ presents probability. Since the following convergence result in probability

then

that is

\[
\lim_{\varepsilon \to 0} \lim_{n \to \infty} D_{m,n}((y_1, y_2)', \varepsilon) = \lim_{\varepsilon \to 0} \lim_{n \to \infty} D_{m,n}(y_1, \varepsilon) + \lim_{\varepsilon \to 0} \lim_{n \to \infty} D_{m,n}(y_2, \varepsilon)
\]
In next section, we will describe how to test nonlinear dependence between \( y_1 \) and \( y_2 \) with (8) and (9). Note that

\[
D_{m,n}(c y, \varepsilon) = \frac{D_{m,n}(y, \varepsilon/c)}{1 + \ln c/\ln(\varepsilon/c)}
\]

\[
\ln c/\ln(\varepsilon/c) \xrightarrow{\varepsilon \to 0} 0
\]

(10)

where cy present series \( \{cy(t), t=1,\ldots,n\} \), \( c \) is a constant. Therefore the estimator of correlation dimension by (5) does not heavily depends on the measurement while \( \varepsilon \) is very small. But in general, we can only have substantially few data from the economic system, so we can not let \( \varepsilon \) to be very small, and (10) does not be satisfied. Therefore the estimator of correlation dimension by (5) depends on the measurement heavily. To solve this problem, we define

\[
D_{m,n}(y, \varepsilon_1, \varepsilon_2) = \frac{n(C_{m,n}(y, \varepsilon_1)/C_{m,n}(y, \varepsilon_2))}{\ln(\varepsilon_1/\varepsilon_2)}
\]

(11)

It is clear that (11) does not heavily depends on the measurement, and also we have

\[
\lim_{\varepsilon_1 \to 0, \varepsilon_2 \to 0} \lim_{n \to \infty} D_{m,n}(y, \varepsilon_1, \varepsilon_2) = \lim_{\varepsilon \to 0} \lim_{n \to \infty} D_{m,n}(y, \varepsilon)
\]

3. The Nonlinear Dependence Coefficient Based on Correlation Dimension

We can make inference about nonlinear dependence between observation variable \( y_1 \) and \( y_2 \) with (8) and (9). If \( D_{m,n}((y_1, y_2'), \varepsilon) \), \( D_{m,n}(y_1, \varepsilon) \), and \( D_{m,n}(y_2, \varepsilon) \) are approximately the same, then we can sure that observation variables \( y_1 \) and \( y_2 \) come from a same dynamical system. In the other word, \( y_1 \) and \( y_2 \) are nonlinear dependent on each other heavily, and should be considered together in forecasting. If \( D_{m,n}((y_1, y_2'), \varepsilon) \) is approximately the same as the sum of \( D_{m,n}(y_1, \varepsilon) \) and \( D_{m,n}(y_2, \varepsilon) \), then we can sure that \( y_1 \) and \( y_2 \) are two irrelative variables. In the other word, \( y_1 \) and \( y_2 \) are nonlinear independent, and they can not provide any information each other in forecasting. However, most economic series are between those two extreme cases. In order to make the above idea more concrete, we recall that correlation coefficient is used generally in statistical theory to describe linear relation between two variables. Therefore, we give the definition of nonlinear dependence coefficient between two series.

**Definition 1** Nonlinear dependence coefficient of series \( \{y_1(t), t = 1,\ldots,n\} \) and \( \{y_2(t), t = 1,\ldots,n\} \) is defined as

\[
R_{m,n}(y_1, y_2, \varepsilon) = \frac{I_{m,n}(y_1, y_2, \varepsilon)}{D_{m,n}((y_1, y_2'), \varepsilon)}
\]

(12)

where

\[
I_{m,n}(y_1, y_2, \varepsilon) = D_{m,n}(y_1, \varepsilon) + D_{m,n}(y_2, \varepsilon) - D_{m,n}((y_1, y_2)', \varepsilon)
\]

It is clear that nonlinear dependence coefficient is 1 if (8) is satisfied, nonlinear dependence coefficient is 0 if (9) is satisfied, and nonlinear dependence coefficient is between 0 and 1 for
other cases. The larger nonlinear dependence coefficient is, the stronger nonlinear dependence is. Therefore it is reasonable to test nonlinear dependence between \( y_1(t) \) and \( y_2(t) \) by nonlinear dependence coefficient. In addition, we have

\[
R_{m,n}(cy_1, cy_2, \varepsilon) = R_{m,n}(y_1, y_2, \varepsilon/c)
\]

(13)

where \( cy_1 \) and \( cy_2 \) present \( \{cy_1(t), t = 1, \ldots, n\} \) and \( \{cy_2(t), t = 1, \cdots, n\} \) respectively, \( c \) is a constant. (13) indicates that nonlinear dependence coefficient given by Definition 1 is stable and does not depends on the measurement even for small data sets.

4. Some Theoretical Results

Here give the proof of the result (6).

**Hypothesis 1** Consider system \((h,F,x(0))\),

1. \( h \) and \( F \) are smooth at least \( C^2 \). \( F \) has a unique compact attractor \( \Lambda \). \( F \) has an orbit which is dense in \( \Lambda \). \( F \) is equipped with a unique ergodic invariant measure \( \rho \) that has a continuous density \( \rho(dx) = j(x)dx \).
2. The forward orbit \( \{x(t), t = 0, 1, \cdots\} \) determined by \( x(0) \) lies on the orbit which is dense in \( \Lambda \).
3. The largest Lyapunov exponent is positive.

**Lemma 1** Consider system \((h,F,x(0))\). Under Hypothesis 1 and \( m > 2 \cdot D(x) + 1 \), there is a constant \( K \) such that

\[
K \cdot \|x(t) - x(s)\| \leq \|y^m_1(t) - y^m_1(s)\| < K^{-1} \cdot \|x(t) - x(s)\|
\]

**Proof** See the proof of Theorem 2.5 in literature [2].

**Theorem 1** Consider system \((h,F,x(0))\). Under Hypothesis 1 and the supremum norm (4), the result (6) holds.

**Proof** From Lemma 1, there are constants \( \{K_i, i = 1, \cdots, p\} \), such that

\[
K_i \cdot \|x(t) - x(s)\| < \|y^m_1(t) - y^m_1(s)\| < K_i^{-1} \cdot \|x(t) - x(s)\|,
\]

\( i = 1, \cdots, p \)

Let

\[
K = \min\{K_i, i = 1, \cdots, p\}.
\]

Under the supremum norm (4), we have

\[
\|y^m(t) - y^m(s)\| = \max_{i=1, \cdots, p} \|y^m_i - y^m(s)\|
\]

\[
K \cdot \|x(t) - x(s)\| < \|y^m(t) - y^m(s)\| < K^{-1} \cdot \|x(t) - x(s)\|
\]

Hence

\[
c_n(x, K\varepsilon) < C_{m,n}(y, \varepsilon) < C_n(x, K^{-1}\varepsilon)
\]
Note that
\[\lim_{\varepsilon \to +0} \lim_{n \to \infty} \ln C_n(x, K\varepsilon) / \ln \varepsilon = \lim_{\varepsilon \to +0} \lim_{n \to \infty} \ln C_n(x, K\varepsilon) / \ln K\varepsilon = \lim_{\varepsilon \to +0} \lim_{n \to \infty} \ln C_n(x, \varepsilon) / \ln \varepsilon\]
Which gives the desired result (6).

**Corollary 1** Consider system \((h, F, x(0))\). Let
\[z_i(t) = y_{k_i}(t) \in \{y_1(t), \ldots, y_r(t)\}, \quad i = 1, \ldots, q\]
\[z(t) = (z_1(t), \ldots, z_q(t))'\]
be some components of \(y(t)\). Under Theorem 1, generically we have
\[\lim_{\varepsilon \to 0} \lim_{n \to \infty} D_{m,n}(z, \varepsilon) = d\]
Corollary 1 gives the method to estimate system dimension with its any dimensional multivariate observation, and shows that the system dimension estimate with its multivariate observation series is the same as that with its univariate observation series.

**Corollary 2** Consider system \((h, F, x(0))\). Under Theorem 1, generically we have
\[\lim_{\varepsilon \to 0} \lim_{n \to \infty} R_{m,n}((y_1, y_2)', \varepsilon) = 1\]
Corollary 2 shows that the nonlinear dependence coefficient between \(y_1(t)\) and \(y_2(t)\) must be asymptotically 1, if \(y_1(t)\) and \(y_2(t)\) come from the same dynamical system with unique compact attractor, and the size of data is large enough.

5. Numerical Results

In this section, by two numerical examples we show that the method proposed in this paper is practical and valid.

**Example 1** Consider the following Henon map\(^5\)
\[x_1(t + 1) = 1 - a \cdot x_1(t)^2 + x_2(t)\]
\[x_2(t + 1) = b \cdot x_1(t)\]
\[y_1(t + 1) = 0.2 \cdot x_2(t + 1) + 0.2 \cdot x_1(t + 1)\]
\[y_2(t + 1) = 0.3 \cdot x_2(t + 1) + 0.3 \cdot x_1(t + 1)\]
where \(y_1\) and \(y_2\) are two observation variables. Let observation data \(\{v(t) : t = 1, \ldots, n\}\) come from the following Logistic map
\[u(t + 1) = A \cdot u(t) \cdot (1 - u(t))\]
\[v(t + 1) = u(t + 1)\]
where the parameters and initial values are

\[ a = 1.4, \quad b = 0.3 \]
\[ x_1(-10000) = x_2(-10000) = 0 \]
\[ A = 4, \quad u(-10000) = 0.1 \]

The initial values begin with -10000, so that the observation data \( y_1(t) : t = 1, \cdots, n \), \( y_2(t) : t = 1, \cdots, n \) and \( v(t) : t = 1, \cdots, n \) are not dependent on the initial values very much. 

\( n = 2000 \). The numerical result based on the equation (11) is described in Table 1.

<table>
<thead>
<tr>
<th>( m )</th>
<th>4</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>6</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_1 )</td>
<td>0.010</td>
<td>0.008</td>
<td>0.010</td>
<td>0.008</td>
<td>0.010</td>
<td>0.008</td>
</tr>
<tr>
<td>( \varepsilon_2 )</td>
<td>0.005</td>
<td>0.004</td>
<td>0.005</td>
<td>0.004</td>
<td>0.005</td>
<td>0.004</td>
</tr>
<tr>
<td>( D_{m,n}(y_1, \varepsilon_1, \varepsilon_2) )</td>
<td>1.1031</td>
<td>1.1262</td>
<td>1.1050</td>
<td>1.1513</td>
<td>1.1210</td>
<td>1.1846</td>
</tr>
<tr>
<td>( D_{m,n}(y_2, \varepsilon_1, \varepsilon_2) )</td>
<td>1.2125</td>
<td>1.1814</td>
<td>1.2126</td>
<td>1.1448</td>
<td>1.1939</td>
<td>1.1313</td>
</tr>
<tr>
<td>( D_{m,n}(y_1,y_2,\varepsilon_1,\varepsilon_2) )</td>
<td>1.1893</td>
<td>1.1642</td>
<td>1.1823</td>
<td>1.1505</td>
<td>1.1621</td>
<td>1.1586</td>
</tr>
<tr>
<td>( R_{m,n}(y_1,y_2,\varepsilon_1,\varepsilon_2) )</td>
<td>0.9470</td>
<td>0.9822</td>
<td>0.9602</td>
<td>0.9957</td>
<td>0.9919</td>
<td>0.9988</td>
</tr>
<tr>
<td>( D_{m,n}(v, \varepsilon_1, \varepsilon_2) )</td>
<td>1.0161</td>
<td>0.9875</td>
<td>1.0996</td>
<td>1.0332</td>
<td>1.1596</td>
<td>1.0783</td>
</tr>
<tr>
<td>( D_{m,n}(y_1,v,\varepsilon_1,\varepsilon_2) )</td>
<td>2.0325</td>
<td>2.0130</td>
<td>2.0440</td>
<td>2.1632</td>
<td>2.1369</td>
<td>2.1535</td>
</tr>
<tr>
<td>( R_{m,n}(y_1,v,\varepsilon_1,\varepsilon_2) )</td>
<td>0.0420</td>
<td>0.0500</td>
<td>0.0785</td>
<td>0.0098</td>
<td>0.0672</td>
<td>0.0508</td>
</tr>
</tbody>
</table>

Simulated results indicate the following facts. Observation variable \( y_1 \) and \( y_2 \) come from a same dynamical system, or in other words \( y_1 \) and \( y_2 \) are heavily nonlinear dependent on each other, so that \( D_{m,n}(y_1,y_2) \), \( D_{m,n}(y_1) \) and \( D_{m,n}(y_2) \) are approximately the same, and the nonlinear dependence coefficient between \( y_1 \) and \( y_2 \) is approximately 1. Observation variable \( y_1 \) and \( v \) come from the different dynamical systems, or in other words \( y_1 \) and \( v \) are independent, so that we have

\[ D_{m,n}(y_1,v) = D_{m,n}(y_1) + D_{m,n}(v) \]

approximately, and the nonlinear dependence coefficient between \( y_1 \) and \( v \) is approximately 0.

**Example 2** The general analysis on real estate by using price index has been carried out for many years in China. The price index is composed by the data from the market research. These data which keep track of the quotations on the market all the times are come from different estates and become a dynamic graph used to observe the quotation on the market. The quantitative research and the precise description and forecast on the orbit track of price index play a major role in real estate research. Here we give the application of our method
about the Shanghai Composite Index of Chinese Restate and Shanghai Second-hand Index as example.

Let $x, y$ present Shanghai Composite Index of Chinese Restate and Shanghai Second-hand Index respectively. The data from January in 2005 to October in 2007 are obtained. The data are deseasonized and normalized first. The numerical result is described in Table 2.

Table 2 The numerical results of Shanghai Composite Index of Chinese Restate and Shanghai Second-hand Index based on the equation (11)

<table>
<thead>
<tr>
<th>$m$</th>
<th>4</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>6</th>
<th>6</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_1$</td>
<td>0.6</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>0.3</td>
<td>0.25</td>
<td>0.3</td>
<td>0.25</td>
<td>0.3</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>$D_{m,n}(y_1, \varepsilon_1, \varepsilon_2)$</td>
<td>1.81</td>
<td>1.71</td>
<td>1.66</td>
<td>1.92</td>
<td>1.82</td>
<td>1.87</td>
<td>1.7983</td>
</tr>
<tr>
<td>$D_{m,n}(y_2, \varepsilon_1, \varepsilon_2)$</td>
<td>1.64</td>
<td>1.59</td>
<td>1.47</td>
<td>1.73</td>
<td>1.66</td>
<td>1.76</td>
<td>1.6416</td>
</tr>
<tr>
<td>$D_{m,n}(y_1, y_2, \varepsilon_1, \varepsilon_2)$</td>
<td>1.89</td>
<td>1.84</td>
<td>1.71</td>
<td>2.02</td>
<td>1.92</td>
<td>1.94</td>
<td>1.8866</td>
</tr>
<tr>
<td>$R_{m,n}(y_1, y_2, \varepsilon_1, \varepsilon_2)$</td>
<td>0.8253</td>
<td>0.7934</td>
<td>0.8304</td>
<td>0.8069</td>
<td>0.8125</td>
<td>0.8711</td>
<td>0.8233</td>
</tr>
</tbody>
</table>

The Nonlinear dependence coefficient of Shanghai Composite Index of Chinese Restate and Shanghai Second-hand Index is about 0.82. It indicates that Shanghai Composite Index of Chinese Restate and Shanghai Second-hand Index are strong nonlinear dependent on each other, they should be considered as a same system in forecasting.

The methods of this paper have been used in variable select to construct nonlinear forecasting model such as neural network\(^{(7)}\), and show very well performance.

6. Conclusions

In this paper, a statistical inference method is proposed. This method can be used in variable selecting for economic forecasting. The equation (6) is the main result of this paper. Although (6) is not difficult to be proved in mathematics, but it gives a most useful idea to test the nonlinear dependence between time series. In addition, Corollary 1 shows that the system dimension estimate with its multivariate observation series is the same as that with its univariate observation series. Numerical results show that the method presented in this paper is effective.

The nonlinear dependence coefficient concept given in this paper is not consummate, and should be perfected gradually in its applications. The applications of chaotic series strange attractors theory, fractal dimension concepts and embedding techniques in forecasting of complex economic systems, need much more research in future. We believe that how to construct forecasting model with strange attractors is more helpful than how to test the nonlinear dependence. The forecasting model based on strange attractors will be the competition to conditional forecasting model.
References:


Initial Value Sensitivity in Technology Diffusion*

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Abstract: Initial value sensitivity in technology diffusion, an important problem for firms’ decision making such as the timing and target market chosen for new technology or product entering the market, has long been limited by the research methodology and tool. Based on the network extension of Bass model, this paper proposes a stochastic threshold model and uses computer simulation to empirically examine three propositions on initial value sensitivity in technology diffusion process. Our findings suggest that diffusion extent is sensitive to not only the number of initial adopters but also their positions in social network, and the variance of customers’ initial assessment as well, which can be detailed as follows: (1) the degree of technology diffusion exhibits highly positive relation to initial adopter quantity, in particular, when the quantity of initial adopters is small, diffusion extent is very sensitive; (2) diffusion extent is sensitive to the positions of initial adopters; (3) in addition, the variance of customers’ initial evaluation displays strong negative relation to the final diffusion degree in that the larger variance, the lower of diffusion extent.

Key words: initial value sensitivity; technology diffusion; social network; computer simulation

1. Introduction

Technology diffusion has long been an attractive problem for scholars. Rogers concludes that the diffusion process consists of four key elements including innovation, communication channels, time and the social system\textsuperscript{[1]}. Among them, communication channels, which are the means by

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which information about an innovation is transmitted to or within the social system, has caught much attention for its importance to technology diffusion. Combining the results of Fourt[2] and Mansfield[3], Bass first points out that world-of-mouth and mass media are two main channels to technology diffusion and then provide a differential equation to display the dynamic change of cumulative adopters[4]. Based on Bass model, the later researchers extend many models such as repeated purchase models[5–7], competitive diffusion models[8,9], and repeated competitive diffusion models[10]. Although these models above all emphasize the importance of two channels as well as the heterogeneity of customer behaviors, there are still many questions needed further research: how mass media and interpersonal communication affect the adopters’ decision? how heterogeneity of participants influence the diffusion process? All these problems concern with extreme large number of persons and complicated social systems which are reluctant to study using traditional empirical methodology like questionnaires, especially in terms of diffusion process considering the variation of consumers’ decision habits.

In this paper, we view the spread of a new technology or product is a kind of complicated process in complex system that may be a dissipation structure having some special characteristics. We try to address the following question: whether and to what extent the technology diffusion depends on the initial value, that is the initial number of adopters and their positions in social network? Since traditional field survey can only provide a realistic situation of diffusion, this method is not suitable to studying the sensitivity of initial vale that needs numerous repeated experiments to compare the variances of the final diffusion extent. Therefore, computer simulation method is used in this article to solve this problem. We first employ complex network method to extend the Bass model from network perspective by means of building a stochastic threshold model from social network perspective and then using numeric simulation to examine the related hypotheses. The main contribution of this paper concentrates on systematic research of the sensitivity of initial value during technological diffusion process. Our findings suggest that technology diffusion is sensitive to not only the number of initial adopters but also their positions in social network, and the variance of customers’ initial evaluations as well.

2. Stochastic Threshold Model Based on Social Network

The most significant contributions of Bass model lie in the combination of two main communication channels and the S-shaped cumulative adopter distribution. Given the basic structure of the Bass model, later scholars try to answer the following questions such as how the diffusion process has been triggered? How it extend to the other individuals within a social system? How the heterogeneity of consumers impact the diffusion extent. These problems are difficult to address because of the complex of diffusion process that generally consists of large number of consumers in the system. In the recent 10 years, some scholars have started to employ method from social network perspective solving diffusion problem. For instance, Harkola
and Greve investigate the impact of network cohesion and structural equivalence through a construction technology diffusing within a Japanese corporate. They suggest that cohesion is significant in high density network while structural equivalence is key to the diffusion in low density network\[11\]. Using a core-periphery network, Abrahamson and Rosenkopf examine how organization network structure affects the diffusion extent\[12\].

In fact, the two information channels word-of-mouth and mass-media in Bass model construct acquaintance network and media network respectively. Each node in mass-media network represents a consumer or a media organization and the connection between two nodes denotes the information channel, through which the assessment of consumer for new technology is influenced. For acquaintance network, all nodes are consumers and their relationships may be kinships, classmates, friends or colleagues and so on, through which information is transferred. Hence it is natural to study technology diffusion from social network perspective. In this paper, we extent Bass model to a stochastic threshold model and Figure 1 illustrates its basic thoughts.

![Schematic graph of stochastic threshold model for technology diffusion](image)

In Figure 1, potential adopters have varying predispositions against adopting an innovation. A potential adopter will give in to a threshold to adopt only if it exceeds a point at which the strength of three aspects, including individual internal assessment of the innovation’s utility external bandwagon pressure coming from media network and acquaintance network, to adopt is greater than the potential adopter’s predisposition against adopting. Therefore, a potential adopter with a high threshold adopts only in response to a strong bandwagon pressure, whereas it only takes a weak bandwagon pressure to cause a potential adopter with a low threshold to adopt, and it takes no bandwagon pressure for a potential adopter with a zero threshold to do so. Potential adopters with zero thresholds will adopt first and then raise the strength of the bandwagon pressure, leading other potential adopters with higher threshold to adopt. Finally when the bandwagon pressure exceeds the highest threshold of potential adopter, all candidates will adopt the innovations and the diffusion is sufficient. The diffusion stops whenever the increase of bandwagon pressure in one cycle of process is insufficient to prompt the non-adopter with the lowest threshold to adopt\[12\]. From Figure 1 we can see, the bandwagon pressure, which
is concerned with the network structure, is significant to the potential adopter’s behaviors. We then express their relation by means of the following equation

\[ B_{i,k} = I_i + a_i S_{i,k-1} + b_i M_{i,k-1} + \varepsilon_{i,k-1} \]  

(1)

where \( B_{i,k} \) is potential adopter \( i \)'s utility assessment of the innovation in bandwagon cycle \( k \). \( I_i \), which represents potential adopter \( i \)'s initial assessment of the innovation’s utility, shows the consumer’s individual evaluation for the new technology or product. \( a_i S_{i,k-1} \) stands for the impact of bandwagon pressure on potential adopter \( i \)'s coming from acquaintance network after \( k-1 \) cycles. Among them, \( S_{i,k-1} \) denotes the impact of bandwagon pressure on potential adopter \( i \). \( a_i \) denotes how much potential adopter \( i \) weights the information represented by acquaintance network. Similarly, \( b_i M_{i,k-1} \) stands for the bandwagon pressure on potential adopter \( i \) coming from media network after \( k-1 \) cycles. Among them, while \( M_{i,k-1} \) denotes the impact of bandwagon pressure on potential adopter \( i \). \( b_i \) denotes how much potential adopter \( i \) weights the information represented by media network. \( \varepsilon_{i,k-1} \) denotes the random factors affecting potential adopter \( i \).

In sum, \( M_{i,k-1} \) and \( S_{i,k-1} \) display the bandwagon pressures on potential adopter \( i \) coming from two networks. \( a_i \) and \( b_i \) describe the extent of potential adopter \( i \)'s dependence on two networks. While \( I_i + b_i M_{i,k-1} \), showing potential adopter \( i \)'s assessment of the technology’s utility, will not be influenced by the other’s opinions, the proportion \( a_i S_{i,k-1} \), representing the bandwagon pressure of adopters on non-adopters, is concerned with the topology position of adopter \( i \) in the acquaintance network.

3. Definition of Initial Value in Technology Diffusion

To be a important characteristic in complex system, sensitivity to initial value represents the phenomenon that initial small disturb will be amplified to generate the large outcome, like exponential growth. Whether the technology diffusion process, a dynamic nonlinear procedure described by Bass model, also exhibits this kind of feature? If so, what on earth the initial value refers to? How to examine it? To what extent final diffusion is influenced by initial value? All the questions is important to diffusion practice when firms try to decide which kind of and how many consumers should be pay more attention to at the very beginning of diffusion.

In Model 1, we define initial value as

\[ B_0 = I + b M_0 + a S_0 \]  

(2)

which can be divided into two parts, one is \( I + b M_0 \), termed as \( SI \)

\[ SI = I + b M_0 \]  

(3)

\( SI \) denotes consumer’s initial assessment of the new technology’s utility, which is independent of other consumers’ decisions. According to limit theorem in probability theory, when the number of consumers are very large, \( SI \) is subject to normal distribution. Thus its variance shows the differences of consumer’s initial assessment and its mean represents the number
of first adopters under the condition of fixed variance. $aS_0$ denotes the bandwagon pressure of adopters in acquaintance network, which is determined by their topology position in the network.

As we discuss above, there are three kinds of definition for initial value named as the number of initial adopters, the typological positions of initial adopters, and consumer’s initial assessment of the new technology’s utility. Because the main purpose of this paper is to examine the sensitivity of initial value, we put forwards a proposition for each of definitions.

**Proposition 1** The extent of technology diffusion is sensitive to the number of initial adopters.

**Proposition 2** The extent of technology diffusion is sensitive to initial adopters’ typological position in social network.

**Proposition 3** The extent of technology diffusion is sensitive to the variance of consumer’s initial assessment of the new technology utility.

To examine the corresponding changes of diffusion extent when facing with the small change of initial value proposed in the three propositions, we introduce elasticity index $E$ as a measurement to illustrate the sensitivity, which is defined as the ratio of the percentage changes of two factors. In the following section, we will first provide the results of numerical simulation and then give some explanations as well.

4. Numerical Simulation

As suggested by Davis, Eisenhardt and Bingham, simulation is an increasing significant methodological approach when the theoretical focus is longitudinal, nonlinear, or processual, especially when empirical data are challenging to obtain\cite{13}. Furthermore, Harrison, Carroll and Carley point out that numerical simulation is particularly useful for examining how sensitive the behavior of a system is to initial conditions\cite{14}. Sensitivity analysis can be solved neither through traditional market investigation method nor observing a real diffusion process of certain technology because it needs not only large number of samples but also understanding the whole process, in particularly, repeated experiments are necessary to testing the sensitivity of initial value. So far, some scholars have applied simulation method to sensitivity analysis. For example, Costantini, Gobet, and Karoui use Monte Carlo method to study the boundary sensitivities for diffusion process in time dependent domains\cite{15}. Therefore, in this paper, we also employ computer simulation to address the sensitivity problems proposed before.

4.1. Network Construction

In our simulation, we construct acquaintance network with 5000 nodes and its average degree is 18, in other words, there are 5000 consumers in the system and each one connects to other 18 persons through various relations like relative, friend, classmate, colleagues and so on. For this kind of social network, recent empirical study suggest that it is a small world with comparatively high clustering and short paths\cite{16,17}. Here we make use of their findings and construct
acquaintance network by the algorithm proposed by Watts\cite{17}. For media network, considering
the decreasing marginal return of advertisement effects on consumer, we assume there are total
3 times for advertisement. The first time is done at the very beginning of diffusion and later
two times are at intervals of certain time, where the impact of each advertisement is subject to
normal distribution. Also we terms the weights of acquaintance and media network as \(a\) and \(b\)
respectively.

4.2. Simulation Results

4.2.1. Test for Proposition 1

For proposition 1, which concerns the sensitivity of the number of initial adopters to the diffusion
extent, we define initial value as the number of initial adopters after first advertisement.

In Figure 2, horizontal axis exhibits the ratio of number of initial adopters from 1 to 150 to
5000 (denoted as \(PLA\)) and the vertical axis illustrates the diffusion extent, the ratio of final
accumulated adopters to 5000. For each initial number, we test 100 times and reach the average
diffusion extent. When \(PLA\) exceed 0.03, that is the initial number is more than 150, \(P\) is above
90\%, representing nearly sufficient diffusion. So it is not necessary to test initial adopters over
150. As shown in Figure 2, the increasing tendency of the diffusion extent with the number of
initial adopter is quite clear. We can explain this phenomenon in two aspects. First, for a fixed
distribution of \(SI\), each number of initial adopters will have a corresponding mean of initial
consumers’ assessment. Therefore, the increase of the number of initial adopters implys that the
total level of consumer’s initial assessment is higher than before; second, more initial adopters
will exert larger bandwagon pressure through social network and make more consumers adopt
new technology. In addition, correlation analysis also demonstrates the very high dependence
of diffusion extent to initial adopters (see Table 1, correlation coefficient =0.997).

For further understanding the sensitivity of initial adopters, we use elasticity index \(E_N\)
defined as \(E_N = \frac{\Delta P\%}{\Delta PLA\%}\) to measure the sensitivity of the diffusion extent to initial adopters.
As shown in Figure 3, when \(P\) is smaller than 0.01, that is initial adopters is less than 50, \(E_N\) is
more than 1, showing that the diffusion extent is elastic to initial number of adopters. However with the increasing of initial adopters until over 100, the elasticity is gradually dropping. When $PLA$ is larger than 0.001, that is initial adopters is more than 50, $E_N$ is less than 1 showing that the diffusion extent is inelastic to initial adopters. Overall, the sensitivity of diffusion extent to initial adopters is higher in the case the number of initial adopters is small while with the increasing of this number, sensitivity is decreasing.

Table 1 Correlation analysis between diffusion extent and initial adopters

<table>
<thead>
<tr>
<th></th>
<th>Spearman’s rho</th>
<th>Diffusion extent</th>
<th>Initial adopters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diffusion extent</td>
<td>Correlation Coefficient</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$N$</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Initial adopters</td>
<td>Correlation Coefficient</td>
<td>0.997(**)</td>
</tr>
<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$N$</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

** Correlation is significant at the 0.01 level (2-tailed).

![Fig. 3 Initial adopters elasticity of diffusion extent](image)

4.2.2. Test for Proposition 2

Proposition 2 focuses on the sensitivity of diffusion extent to typology positions of initial adopters given the number of them. Here in this paper, we first test the proposition with fixed number of initial adopter 52, and then we generalize the results to other values. For the extreme complexity of social network, it is nearly impossible to identify the exact typological position of each adopter. So we employ some indexes like clustering coefficient $C_i$ and degree $K_i$ to roughly describe adopter $i$’s position in the network[18]. The bottom left panel (Figure 4) shows no apparent relationship between diffusion extent and clustering coefficient $C_i$ and the same result can be obtained from the bottom right panel (Figure 5) for average degree $K_i$, where for each index we test for 1000 times.

In this paper, according to our threshold model, we construct a new statistical index $BP$ defined as

$$BP = \sum_{i} \frac{a_i}{1 - B_{i,0}} \times \frac{1}{k_i}$$

(4)
where $i$ represents the consumer who has direct linkage with initial adopter; $a_i$ denotes the weight of consumer $i$ to the acquaintance network. As we defined in the equation (2), $B_{i,0}$ denotes consumer $i$’s initial utility. Since the position of initial adopter can be identified by $k_i$, $a_i$ and $B_{i,0}$, we may depict the position of initial adopter by using $BP$ that is the function of all $k_i$, $a_i$ and $B_{i,0}$, showing the bandwagon pressure of initial adopters on nonadopters connecting to them.

As we can see from the scatter diagram above (Figure 6), there exists a positive relationship where diffusion extent tends to increase as $BP$ increases. The large $BP$ means higher bandwagon pressure of initial adopters on potential adopters through positive feedback mechanism between two networks. In addition, correlation analysis also indicates a significant positive relationship between diffusion extent and $BP$ index (see Table 2, correlation coefficient =0.45).

Similar to $E_N$, we use elasticity index $E_{BP}$ defined as $E_{BP} = \frac{\Delta P\%}{\Delta BP\%}$ to further measure the sensitivity. As shown in Figure 7, few points are between lines -1 and 1, indicating that the positions of initial adopters are elastic to diffusion extent. Then we generalize our findings above by using other initial number of adopters and repeated the examining process above. The similar results indicate that the diffusion extent is sensitive to the typological position of initial adopters.
Fig. 6 The sensitivity of $BP$ structural index to diffusion extent

Table 2 Correlation analysis between diffusion extent and $BP$ index

<table>
<thead>
<tr>
<th></th>
<th>$Bp$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Bp$ Spearman’s rho</td>
<td>Correlation Coefficient</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$N$</td>
<td>1000</td>
</tr>
<tr>
<td>$P$</td>
<td>Correlation Coefficient</td>
<td>0.450(***)</td>
</tr>
<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>$N$</td>
<td>1000</td>
</tr>
</tbody>
</table>

** Correlation is significant at the 0.01 level (2-tailed).

Fig. 7 $Bp$ elasticity of diffusion extent

4.2.3. Results of Proposition 3

For Proposition 3, which concerns the sensitivity of initial variance of consumers’ assessments to the diffusion extent, as we mentioned before, the initial value here refers to consumers’ heterogeneity on initial evolution of technology or product. We use variance termed as $\sigma$ to measure their differences. Figure 8 illustrates the relationship between diffusion extent and initial variance of consumers’ assessments of new technology.

For each $\sigma$, we test for 100 times and get their average value, given the number of initial adopters, the variance and mean of consumers’ assessments are corresponding one by one. Large variance lowers the consumers’ assessment level, making them more difficult to adopt
new technology. As shown in Figure 8, there is a strong negative relationship between $\sigma$ and $P$, indicating that heterogeneity of consumers is one of important factors effecting technology diffusion. In addition, correlation analysis shows a negative relationship between diffusion extent and initial assessment variance (see Table 3, correlation coefficient = $-0.821$).

![Fig. 8 Relationship between diffusion extent and initial variance of consumers' assessments (initial adopter ratio= 0.26%)](image)

**Table 3** Correlation analysis of diffusion extent and initial assessment variances

<table>
<thead>
<tr>
<th>sigma</th>
<th>$\rho$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Coefficient</td>
<td>1.000</td>
<td>$-0.821^{(**)}$</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.</td>
<td>0.000</td>
</tr>
<tr>
<td>$N$</td>
<td>10100</td>
<td>10100</td>
</tr>
</tbody>
</table>

$^{(**)}$ Correlation is significant at the 0.01 level (2-tailed).

To further measure the sensitivity of $\sigma$ to diffusion extent, we introduce $E_{\sigma}$ index, which is defined as $E_{\sigma} = \frac{\Delta P\%}{\Delta \sigma\%}$. As illustrated in Figure 9, when $\sigma$ is between 5 and 30, the absolute of $E_{\sigma}$ is more than 1, indicating $\sigma$ is elastic to diffusion extent. With the increasing of $\sigma$, most of $E_{\sigma}$ are between $-1$ and $1$, showing that diffusion extent is not very sensitive to the change of variance of initial adopters’ assessments of new technology.

5. Conclusion and Further Research

This paper attempts to address the sensitivity of initial conditions to diffusion extent by using simulation approach from complex network perspective. Based on the two main communication channels proposed by Bass, we first establish a stochastic threshold model that highlights the importance of bandwagon pressures coming from media and acquaintance networks and consumers’ initial assessment as well. Then making use of simulation method, we examine three propositions and our simulation results suggest that (1) diffusion extent exhibits significant
positive relation with the number of initial adopters, in particular, diffusion extent is comparatively more sensitive to small number of initial adopters; (2) diffusion extent is sensitive to the positions of initial adopters; (3) in addition, the variance of customers’ initial assessment displays strong negative relation to the final diffusion extent where large variance will lower the diffusion extent. In conclusion, diffusion extent is sensitive to initial values in terms of the number and the typological position of initial adopters, and variance of consumers’ initial assessment of new technology’s utility.

The practical applications of sensitivity problem for firm’s decision lie in the following area: first, since diffusion extent is highly sensitive to the number and typological positions of initial adopters, the initial conditions such as the enter timing of new technology or product, first advertisement, target market chosen etc will set significant impact on diffusion. Who the initial adopters are and how many of them after first advertisement and how about their positions in social network should be taken seriously in marketing strategy. Adopters with extensive linkages will help to prompt the diffusion process. Second, the sensitivity of diffusion extent to the variance of consumer’s initial assessment may indicate that firms should consider how to narrow the differences among various consumers. One example is to warm the market before the entry of new product. At that time, the main purpose of advertising is to create similar image or understanding for the new product and to build solid ground for the future diffusion.

Since the precondition of this paper is a positive evaluation to new technology or product, which can create a positive feedback through the bandwagon pressures coming from acquaintance network and media network, and the interaction between two networks as well. This positive evaluation implies that the product could satisfy the need of consumers. However, in practice, mixed evaluations including negative and positive ones should be more common. Both of them can create bandwagon pressure and spread through social network. One of the future directions is to combine these two kinds of evaluations and test the sensitivity problems mentioned above. Meanwhile in this paper the acquaintance network is a small-world. Yet in the real world, many diffusions of new technology or product happen in the system with different network structure. Further research can examine the sensitivity problems in the other
types of network such as scale-free network or Poisson distribution network.

References:

A Credit Risk Evaluation Approach to Neural Network Training by Means of Financial Ratios

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Abstract: In recent years artificial neural networks are used to recognize the risk category of investigated companies. The research is based on data from 81 listed enterprises that applied for credit in domestic regional banks operating in China. The backpropagation algorithm-the multilayer feedforward network structure is described. Each firm is described by 9 diagnostic variables and potential borrowers are classified into four classes. The efficiency of classification is evaluated in terms of classification errors calculated from the actual classification made by the credit officers. The results of the experiments show that Levenberg-Marquardt training error is smallest among 4 learning algorithms and its performance is better, and application of artificial neural networks and classification functions can support the creditworthiness evaluation of borrowers.

Key words: credit risk evaluation; financial ratio; neural network; classification algorithms; the multilayer network

1. Introduction

Credit risk analysis is an important topic in the financial risk management. Due to recent financial crises and regulatory concern of Basel II, credit risk analysis has been the major focus of financial and banking industry(Zhi H., Li M., 2005)[1]. Financial institutions such as banks, leasing companies, investment and pension funds are subject to financial risk. The main risks are: credit, market, liquidity, and operational. To reduce credit risk, financial institutions perform an economic analysis of each potential borrower. This is realized through studying a map that projects a multidimensional space of economic performance indicators (turnover, profit, credit history, and the like) into a binary accomplishment set of decisions about allotting credit(Yuan Z., 1999)[2].

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Traditionally, credits are granted based on a judgmental concept using past experiences of the credit officers. This approach suffers, however, from: high costs of training credit officers; frequent incorrect decisions made by credit officers; the long period of time that is required to evaluate the risk category of the client and to make the credit granting decision; and different decisions (made by different credit officers) for the same case. These difficulties suggest the need to automate credit management decisions.

Therefore, in the last 30 years, there has been a substantial development in the area of applying mathematical methods to finance. An alternative approach, proposed by Altman (1968, 1988 and 1993), is based on classical statistical classification tools like discriminant analysis. One can also consider the application of artificial neural networks (ANN) and genetic algorithms to solve the problem of borrower classification and bankruptcy predictions, as discussed by Rahimian et al. (1993), Odom and Sharda (1993), Raghupathi et al. (1993), Wilson and Sharda (1994), Baetge and Krause (1993), Berry and Trigueiros (1993), Lacher et al. (1995), Tam and Kiang (1990, 1993), Back et al. (1995), Rehkugler and Poddig (1992), Witkowska (1999), Witkowska and Staniec (2003), and Witkowska et al. (2004).[3,4]

The different types of classification algorithms for credit scoring systems most usually used to evaluate credit risk are: logistic regression, linear and quadratic discriminant analysis; Linear programming; support vector machines; neural networks; bayesian network classifiers; decision trees and rules; K-Nearest neighbor classifiers. They have strengths and weaknesses. However, artificial neural networks have been introduced to evaluate credit scoring systems more objectively and consistently (Simon, 2001).[5]

Neural Network-based systems that allow the system, through an analysis of historical data, to determine the relationship between account characteristics and the probability of default. In many instances, they are more flexible than standard statistical techniques, as no assumptions are made about the functional form of the relationship between factors and the probability of default, or about the distribution of the variables or model error (Zekic, 2006).[6] However, like systems above, artificial intelligence algorithms exist some shortcomings.

In this paper, we study and compare the performance of various state-of-the-art classification algorithms applied to nine real-life financial ratios data sets.

2. Classification Algorithms with Neural Network Model

Backpropagation was created by generalizing the Widrow-Hoff learning rule to multiple-layer networks and nonlinear differentiable transfer functions. Input vectors and the corresponding target vectors are used to train a network until it can approximate a function, associate input vectors with specific output vectors. Networks with biases, a sigmoid layer, and a linear output layer are capable of approximating any function with a finite number of discontinuities.

Standard backpropagation is a gradient descent algorithm, in which the network weights
are moved along the negative of the gradient of the performance function. There are a number of variations on the basic algorithm that are based on other standard optimization techniques, such as conjugate gradient and Levenberg-Marquardt methods. This paper studies the advantages and disadvantages of each algorithm, and discusses preprocessing and postprocessing techniques, which can improve the efficiency and generalization of network training. We use the backpropagation training functions in the MATLAB toolbox and NeuroSolutions to train feedforward neural networks to classify 82 selected listed companies from China.

2.1. Architecture

The backpropagation algorithm—the multilayer feedforward network feedforward network structure is described as follows.

An elementary neuron with $N$ inputs is shown below. Each input is weighted with an appropriate $w$. The sum of the weighted inputs and the bias forms the input to the transfer function $f$. Neurons may use any differentiable transfer function $f$ to generate their output.

Multilayer networks often use the log-sigmoid transfer function. The function logsig generates outputs between 0 and 1 as the neuron’s net input goes from negative to positive infinity. Such nonlinear elements provide a network with the ability to make soft decisions.

$$f(x_i, w_i) = \frac{1}{1 + \exp[-x_{lin}^i]}$$

(1)

where $e^{x_{lin}^i} = \beta x_i$ is the scaled and offset activity inherited from the LinearAxon.

The Weights access point of the SigmoidAxon provides access to the Bias vector.

Alternatively, multilayer networks may use the tan-sigmoid transfer function.

The TanhAxon applies a bias and tanh function to each neuron in the layer. This will squash the range of each neuron in the layer to between -1 and 1. Such nonlinear elements provide a network with the ability to make soft decisions.

Activation Function:

$$f(x_i, w_i) = \tanh[x_{lin}^i]$$

(2)

where $x_{lin}^i = \beta x_i$ is the scaled and offset activity inherited from the LinearAxon.

Supervised learning requires a metric, a measure of how the network is doing.

Components of the backprop plane are responsible for computing the weight gradients and backpropagating the sensitivities. ErrorCriteria components are responsible for determining the error used for the backpropagation.

Components in the ErrorCriteria are defined by a cost function of the form:

$$J(t) = \sum_i f(d_i(t), y_i(t))$$

(3)

and error function:
\( \varepsilon_j(t) = \frac{\partial J}{\partial y_j(t)} \)  

Where \( d_i(t) \) and \( y_i(t) \) are the desired response and network's output, respectively.

ConjugateGradient gradient descent component uses the “scaled conjugate gradient” learning algorithm. It is a member of a class of learning algorithms called “second order methods”.

The conjugate gradient method can move to the minimum of a \( N \)-dimensional quadratic function in \( N \) steps. By always updating the weights in a direction that is conjugate to all past movements in the gradient, you can avoid all of the zig-zagging of 1st order gradient descent methods. Weight Update Equations are:

\[
\Delta w = \alpha(n)p(n) \\
P(0) = -g(0) \\
p(n+1) = -g(n+1) + \beta(n)p(n) \\
\beta(n) = \frac{g^T(n+1)g(n)}{g^T(n+1)g(n)} \\
\alpha(n) = \arg \min(E(w(n) + \eta p(n))
\]

where \( w \) is the weight, \( p \) is the current direction of weight movement, \( g \) is the gradient (backprop information), \( b \) is a parameter that determines how much of the past direction is mixed with the gradient to form the new conjugate direction. The equations are linear search to find the minimum MSE along the direction \( p \).

The Scaled Conjugate Gradient method (SCG) is the method used by NeuroSolutions and it avoids the line search procedure. The algorithm is based on computing \( Hd \) where \( d \) is a vector.

The Levenberg-Marquardt (LM) algorithm is one of the most appropriate higher-order adaptive algorithms known for minimizing the MSE of a neural network.

Like the quasi-Newton methods, the Levenberg-Marquardt algorithm was designed to approach second-order training speed without having to compute the Hessian matrix. When the performance function has the form of a sum of squares, then the Hessian matrix can be approximated as

\[
H = J^T J
\]

and the gradient can be computed as \( g = J^T e \).

where \( J \) is the Jacobian matrix that contains first derivatives of the network errors with respect to the weights and biases, and \( e \) is a vector of network errors. The Jacobian matrix can be computed through a standard backpropagation technique that is much less complex than computing the Hessian matrix.

The Levenberg-Marquardt algorithm uses this approximation to the Hessian matrix in the following Newton-like update:

\[
x_{k+1} = x_k - [J^T J + \mu I]^{-1} J^T e
\]
When the scalar $\mu$ is zero, this is just Newton's method, using the approximate Hessian matrix. When $\mu$ is large, this becomes gradient descent with a small step size. Newton's method is faster and more accurate near an error minimum, so the aim is to shift towards Newton's method as quickly as possible. Thus, $\mu$ is decreased after each successful step (reduction in performance function) and is increased only when a tentative step would increase the performance function. In this way, the performance function will always be reduced at each iteration of the algorithm.

3. **Training and Modeling on Samples From Listed Companies in China**

### 3.1. The Sample Data From Listed Companies

For the empirical analysis we use a database provided by official reports on China Securities Regulatory Commission website which contains quarterly and semiyearly data and yearly data for up to 1363 firms in the years from 1991 to 2006. The samples of this research come from 81 Securities Issuing in Zhejiang province that have been listed by China Securities Regulatory Commission website and the Great Wall stock trading system (www.cc168.com.cn). The data draw from the financial data of the third quarter of 2005 on the website. Because the quality of the business enterprise financial standing depends on whether they can pay principal and interest on time or not, we will carry on the classification to these companies to judge its credit risk level and default only by their finance states. So banks make decision on its credit size and lending rate levels and take it reference as developing the credit risk model. Because the finance index sign is numerous, we use matlab and NeuroSolution to carry on preprocessing data, then adopt 9 financial ratio index to measure the business enterprise characteristics: Liquidity Ratio; Cash Ratio; Equity to Assets Ratio; Inventory Turnover; Assets Liabilities Ratio; Long-Term Liabilities to Assets Ratio; Account Receivable Turnover Rate; Rate of Profit on Net Sales; Return on Total Assets.

We will divide 81 firms data on financial ratios into three groups. One group consists of 40 firms data which is used to train; another consists of 20 firms data which are selected from rest 41 firms data. They are used to test the training model.

### 3.2. Training Processing

Training processing can be divided nine steps. The first step to designing any neural network is to collect training data. Use the “Browse” button to select an input file.

The second step Desired. This network requires supervised training, and we open another file which contains the desired response data and tag it as “Desired”.

Third step Cross Val. & Test Data. This panel allows you to specify both the cross validation and testing data sets. The data can either be extracted from the existing training
Cross validation is a highly recommended method for stopping network training. This method monitors the error on an independent set of data and stops training when this error begins to increase. We build a testing data with 20 firms. The testing set is used to test the performance of the network. Once the network is trained, the weights are then frozen. The testing set is fed into the network and the network output is compared with the desired output.

Fourth step  Multilayer. We set parameters for this step as follow(see Figure 2).

Multilayer perceptrons (MLPs) are layered feedforward networks typically trained with static backpropagation. Here you simply specify the number of hidden layers. These networks have found their way into countless applications requiring static pattern classification. Their main advantages are that they are easy to use, and that they can approximate any input/output map. The key disadvantages are that they train slowly, and require lots of training data (typically three times more training samples than network weights).

Fig.1 Multilayer perceptrons (MLPs)  Fig.2. Multilayer pattern

The fifth step  Hidden Layer 1. Association parameters is exhibited below.

Processing 4 Transfer: sigmoidAxon; Learning: momentum. Step size: 1.00000; Momentum: 0.70000.

This panel is used to specify the parameters a layer of processing elements (PEs). Each layer contains a vector of PEs and that the parameters selected apply to the entire vector. The parameters are dependent on the neural model, but all require a nonlinearity function to specify the behavior of the PEs. In addition, each layer has an associated learning rule and learning parameters.

Learning from the data is the essence of neurocomputing. Every PE that has an adaptive parameter must change it according to some prespecified procedure. Here it is sufficient to say that the weights are changed based on their previous value and a correction term. The learning rule is the means by which the correction term is specified. If the learning rate is too small, then learning takes a long time. On the other hand, if it is set too high, then the adaptation
diverges and the weights are unusable.

The sixth step Output Layer. Transfer: SigmoiAxonbw. Learning: momentum; Step size: 0.100000; momentum: 0.700000.

The seventh step Supervised Learning; Maximum Epochs: 1000; Termination: MSE. Threshold: 0.01; Minimum. Weight update: on-line.

The Maximum Epochs field specifies how many iterations (over the training set) will be done if no other criterion kicks in. The Error Change box contains the parameters used to terminate the training based on mean squared error.

The eighth step Probe Configuration. Performance measures.

4. Result and Discussion

4.1. Experimental Result

We select randomly 40 firms as training samples. According to The Multilayer perceptrons algorithm above, input data and target data are tagged. Then we use matlab and NeuroSolution to train these data respectively.

The following figure 3 is based on the result for training. Model Parameters characteristics are shown as follows.


Figure 3 shows that training performance is 0.253447.

The network topology includes weights to layer and bias to layer.

![Fig.3 Training performance](image)

Iw\{1,1\} is represented for weight to layer 1 from input 1: [0.040951, -2.135, 0.24121, -2.3401, -3.0268, 1.6287, -1.1336, -0.34943, 6.6071]; Iw\{2,1\} is represented for weight to layer: [-10.6931,
4.2246]; b\{1\} is represented for bias to layer 1; b\{2\} bias to layer [11.1152; 4.2051].

4.2. Varying Network Parameters

We compare four different ways of parameterizations. That is, we vary a parameter training process so as to train a neural network multiple times, include different levels of information, estimate the respective parameters.

![Graph](image)

Fig. 4 and Fig. 5 Active performance and cost under LevenbergMarque (On-line Batch)

We will use the “vary a parameter” process to determine the optimum number of hidden layer processing elements, and Sensitivity Analysis and so on. First we adopt Elman backprop as network type. After training performance is 0.180511.

Second we vary learning algorithm. Figures 4–7 show their performances. We find that LevenbergMarque training error is smallest among 4 learning algorithms and its performance is better. Although conjugateGradi converge to stability, its error is up to 24.06%.

Third when we increasing the number of processing elements to 2 or 3, training performance sometimes result in minor improvement. But under some conditions it exists unstable during training (see Figure 7).

![Graph](image)

Fig. 6 Active performance with DeltaBarDelta

![Graph](image)

Fig. 7 Training cost and performance with 2 hidden layers

If we can give insights into the underlying relationships between output layer and hidden layer. Moreover, hidden layer is characteristic of transfer function with TanhAxon and Learning rule with LM; as transfer function and learning rule of output layer are sigmoidAxon and LM respectively. Its error is 3.84%.

Active performance is rather ideal. So we build a new credit classification model.
We select 20 firms from our data samples randomly to test the new building model. Figure 9 presents active performance.

There are generally four steps in the training process: assemble the training data; create the network object; train the network; simulate the network response to new inputs.

Fig. 8 Transfer function with TanhAxon

Fig. 9 Testing performance of the training model response to new inputs

5. Conclusion

As the listed companies are the high-quality customers of banks, it is natural for each bank always compete for. However, because of heavy competition in product market, and difference in the company’s management standard level and its strategic marketing, the performance and potentials of the listed companies exist inconsistent. As a result, the banks should classify their customers, before they provide financial services and make a price to carry on the strategy with difference.

The Neural Network algorithm introduced in this paper is a kind of model for classification more maturely, and has application in a lot of fields. During “training” the neural network fits a system of weights to each financial variable included in a database consisting of nine financial ratios. However, the network may be “overfit” to a particular database if excessive training has taken place, thereby resulting in poor out-of-sample estimates. Moreover, neural networks are costly to implement and maintain. we analyze indicators related to financial attributes, and choose 9 finance indicators. According to better valuation on the listed companies of Zhejiang province, we adopt “try and error” to train model.

From simulation results, we find that multilayer perceptrons can solve for classify credit customers questions. By varying Network Parameters we demonstrate that LevenbergMarque training error is smallest among 4 learning algorithms and its performance is better. Although conjugateGradi converge to stablity, during training its error is too big up to 24.06%.

Increasing the number of hidden layer can result in minor improvement, if we pay attention to relationships between output layer and hidden layer, and adopt association parameters to train model. Active performance is rather ideal. A new credit classification model can have valuable.
References:


Control and Synchronization of a Novel Hyperchaotic System Containing One Quadratic Term

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Abstract: A novel hyperchaotic system derived from Liu system is proposed in this paper. Lyapunov exponent, phase portrait and Poincaré mapping are given to verify that the system is hyperchaotic. A controller is designed to compel the hyperchaotic system to converge into the equilibrium. It is proved theoretically that this control law is feasible and valid by Lyapunov second method. Based on linear feedback synchronization control principle, synchronization control of the novel hyperchaotic system is realized. Numerical simulation shows that this synchronization method is simple and effective. As long as the proper linear feedback control vector is chosen, it is easy to achieve the rapid synchronization between the driving system and response system.

Key words: Liu chaotic system; hyperchaotic system; Poincaré mapping; nonlinear control; linear feedback synchronization

1. Introduction

The engineering value of chaotic system in nonlinear circuit, communication and other information science has been of broad interest in the last three decades. In general, a hyperchaotic system is defined as a chaotic system with more than one positive exponents, implying that its dynamics extend in several different directions simultaneously. Therefore, hyperchaotic systems have more complex dynamical behaviours which can be used to improve the security of chaotic communication systems. Therefore, the topic of theoretical design, control, synchronization

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and circuitry realization of various hyperchaotic signals has recently become hotspots in the research area\cite{1-8}.

Usually, synchronization means the phase trajectories distance $\| x - y \|$ shrink into zero when the time $t \to \infty$. People are searching for synchronization method between hyperchaotic systems at all times. Various effective methods which are applied to make two identical or different chaotic systems up to synchronization are proposed and confirmed since PC method has been advanced. For example, sliding mode variable structure control\cite{9}, backstepping control\cite{10}, adaptive fuzzy tracking control\cite{11}, linear feedback control\cite{12-14} and so on. Each method has its character and fits to its certain area. Among them, the linear feedback control is especially attractive and has been commonly applied to practical implementation due to its simplicity in configuration and implementation. As we know, it is not easy to design the appropriate Lyapunov function in order to compute the range of satisfied control parameter based on the Lyapunov stability theorem to synchronize two chaotic systems. In some sense, it has some more complex mathematical derivation. Furthermore, in some chaotic systems, it is too difficult to design the right Lyapunov function, especially in some piecewise chaotic systems.

Recently, Wang construct a new hyperchaotic system based on Liu system\cite{2}, and design an analog circuit to implement this system. When we substitute the square term with absolute term, the new system is also hyperchaotic and displays abundant dynamical behaviour, Lyapunov exponent spectrum and Poincaré mapping verify the hyperchaotic characteristic. In this paper, a new nonlinear controller is designed which can compel the system converge into equilibrium, and the efficiency is proved theoretically by using Lyapunov second law. Based on the characteristic of linear feedback synchronization control, applying the principle of linear feedback control, simple and yet easily verified sufficient feedback controller is established for chaos synchronization between driving system and response system. Simulation results show that the way works well.

2. New Hyperchaotic System Derived from Liu System

The Liu chaotic system can be given by

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = bx - kxz \\ \dot{z} = -cz + hx^2 \end{cases} \tag{1}$$

Changing the square term to be absolute term, and introduce a state feedback control, we get a new four-dimensional system only containing one quadratic term,

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = bx - kxz + w \\ \dot{z} = -cz + |x| \\ \dot{w} = -gx \end{cases} \tag{2}$$
where \( a, b, c, g, k, h \) are constant, and \( x, y, z, w \) are state variables of the system. When \( a = 10, b = 40, c = 2.5, k = 16 \) and \( g = 9 \), hyperchaotic attractors are as shown in Fig.1. Poincaré mapping yields those points of the confusion as shown in Fig.2. Using Jacobian method to calculate Lyapunov exponents, so we get \( LE_1 = 0.8198, LE_2 = 0.1344, LE_3 = -0.0581, LE_4 = -13.3907 \).

Fig.1 Phase portraits of hyperchaotic system (2) (a) \( x - y \); (b) \( x - z \); (c) \( y - z \); (d) \( x - w \); (e) \( y - w \); (f) \( z - w \).

Two positive values of Lyapunov exponents show that the system is hyperchaotic. It is easy
to see that system (2) has a natural symmetry under the following transformation:

$$(x, y, z, w) \rightarrow (-x, -y, z, -w)$$

Fig. 2 Poincaré map of $x-y$ plane

That is, the system (2) is symmetry about $z$-axis. The equilibrium of the system (2) satisfies the following equations:

$$a(y - x) = 0, \quad bx - kxz + w = 0, \quad |x| - cz = 0, \quad \text{and} \quad gx = 0$$

(4)

Obviously, $s_0 = (0, 0, 0, 0)$ is the unique equilibrium. Doing linearization of the system (2) at the equilibrium $s_0$, so we get Jacobian matrix as:

$$J = \begin{bmatrix}
-a & a & 0 & 0 \\
b - kx & 0 & -kx & 1 \\
\text{sgn}(x) & 0 & -c & 0 \\
-g & 0 & 0 & 0
\end{bmatrix}$$

(5)

For the equilibrium $s_0$, the characteristic equation is

$$f(\lambda) = \lambda^2(\lambda + a)(\lambda + c) = 0$$

(6)

It is easy to draw the conclusion that the equilibrium $s_0$ is unstable according to the Routh-Hurwitz condition.

3. Control of the Hyperchaotic System

3.1. The Controller Design

Design control law, $U = (\mu_1, \mu_2, \mu_3, \mu_4)^T$, and add it to the controlled system (2)

$$\begin{cases}
\dot{x} = a(y - x) + \mu_1 \\
\dot{y} = bx - kxz + w + \mu_2 \\
\dot{z} = -cz + |x| + \mu_3 \\
\dot{w} = -gx + \mu_4
\end{cases}$$

(7)
Theorem 3.1 For the system (2), when control law $U = (\mu_1, \mu_2, \mu_3, \mu_4)^T = \begin{cases} 
 kx - by \\
 -ax \\
 -|x| + kxy \\
 gx - y 
\end{cases}$

$(k$ is feedback gain, $k \leq a$), the system (2) is stable on equilibrium.

**Proof** Using Lyapunov second method, construct positive Lyapunov function, $V = x^2 + y^2 + z^2 + w^2$. For the system (7), it yields that

$$\dot{V} = 2x \dot{x} + 2y \dot{y} + 2z \dot{z} + 2w \dot{w}$$

$$= 2x(ay - ax + \mu_1) + 2y(bx - kxz + w + \mu_2) + 2z(-cz + |x| + \mu_3) + 2w(-gx + \mu_4) \quad (8)$$

substituting with $\mu_1 = kx - by, \mu_2 = -ax, \mu_3 = -|x| + kxy, \mu_4 = gx - y$, so it yields

$$\dot{V} = (2k - 2a)x^2 - 2cz^2 \quad (9)$$

According to the Lyapunov second method, to make the system (7) stable on zero equilibrium, $\dot{V}$ should be negative, so $2k - 2a \leq 0$, i.e. $k \leq a$, so the system (7) is stable on zero equilibrium. It means that when $\mu_1, \mu_2, \mu_3, \mu_4$ and $k$ satisfy the condition of Theorem 3.1, the system can be controlled to stay on zero equilibrium.

3.2. Numerical Simulation

Using matlab software to simulate, the above control method is proved to be valid. The initial point is set as $(2.2, 2.2, 0, 0)$, the feedback gain be set as $k=0$, the stable trajectory of the system (7) as shown in Fig.3. From the figure, we can see that the phase trajectory will converge into zero equilibrium finally and when the control law is satisfied.

![Fig.3 Phase trajectory of the system (7) (a) x - y - z; (b) y - z - w](image)

4. Synchronization of Hyperchaotic System

4.1. Introduction to Linear Feedback Synchronization Principle
Suppose an autonomous chaotic system which is regarded as a drive system:

\[ \dot{X} = F(X) \]  

(10)

The response system is described as:

\[ \dot{Y} = F(Y) \]  

(11)

Add the linear feedback controller to the response system, so the equations will be changed as:

\[
\begin{cases} 
\dot{X} = F(X) \\
\dot{Y} = F(Y) + K(X - Y)
\end{cases}
\]  

(12)

Define error vector as \( E = X - Y \), the equation (12) can be changed as:

\[
\begin{cases} 
\dot{X} = F(X) \\
\dot{E} = G(X, Y, K)
\end{cases}
\]  

(13)

where \( F = (f_1, f_2, \ldots, f_n)^T \), \( G = (g_1, g_2, \ldots, g_n)^T \), \( X = (x_1, x_2, \ldots, x_n)^T \), \( Y = (y_1, y_2, \ldots, y_n)^T \), \( E = (e_1, e_2, \ldots, e_n)^T \) are \( n \)-dimension vectors, \( K = \text{diag}[k_1, k_2, \ldots, k_n]^T \) is linear feedback control parameter vector. \( T \) is transposition. The largest Lyapunov exponent of the system (10) is supposed as \( \lambda_{\text{max}} = \max(\lambda_1, \lambda_2, \ldots, \lambda_n) \) and the value of linear feedback control parameter is defined as \( k = k_1 = \cdots = k_n \). The synchronization aims to design the control parameter which synchronizes the states of both the drive and response system.

**Theorem 4.1**

When the linear feedback control parameter is greater than the largest Lyapunov exponent of the chaotic system, i.e., \( k > \lambda_{\text{max}} \), the two identical chaotic systems with different initial conditions will be synchronized under the linear feedback control.

**Proof**

Vector field divergence of the system (10) and the system (11) can be defined as:

\[ D(X) = \sum_{i=1}^{n} \frac{\partial f_i}{\partial x_i}, \quad D(Y) = \sum_{i=1}^{n} \frac{\partial f_i}{\partial y_i} \]  

(14)

The Lyapunov exponents of the drive and response system are \( \lambda_i(X) \), \( \lambda_i(Y) \), respectively. When the relationship between the vector field divergence and the sum of the whole Lyapunov exponents of the system are considered, the following equations can be obtained:

\[ D(X) = D(Y) = \sum_{i=1}^{n} \lambda_i(X) = \sum_{i=1}^{n} \lambda_i(Y) \]  

(15)

For the system (12),

\[ D = D(X) + D(Y) - \sum_{i=1}^{n} k_i \]  

(16)

For the system (13),

\[ D = D(X) + D(E) \]  

(17)
where $D(E) = \sum_{i=1}^{n} \lambda_i(E) = \sum_{i=1}^{n} \frac{\partial g_i}{\partial e_i}$ is the vector field divergence of the error dynamical system and $\lambda_i(E)$ is its Lyapunov exponent. So, from the equations (15)–(17) a new equation can be obtained

$$D(E) = D(X) - \sum_{i=1}^{n} k_i$$  \hspace{1cm} (18)

So long as $D(E) < 0$, $D(X) - \sum_{i=1}^{n} k_i < 0$, viz., when $k > \lambda_{\text{max}}$, the state of the error dynamical system will be stable at the zero equilibrium point. So, synchronization of the two identical systems with different initial conditions will be achieved.

### 4.2. Synchronization Control Gain Vector Design and Numerical Simulation

In this section, we apply this linear feedback controller to the proposed hyperchaotic system. When the linear feedback controller is added to the driving system (2), it yields the following response system,

$$\begin{align*}
\dot{x}' &= a(y' - x') - k_1(x' - x) \\
\dot{y}' &= bx' - kxz' + w' - k_2(y' - y) \\
\dot{z}' &= -cz' + |x'| - k_3(z' - z) \\
\dot{w}' &= -gx' - k_4(w' - w)
\end{align*}$$  \hspace{1cm} (19)

where $k = k_1 = k_2 = k_3 = k_4$, and $e_1 = x' - x$, $e_2 = y' - y$, $e_3 = z' - z$, $e_4 = w' - w$ are defined. We subtract the system (2) from the system (19) and get the error equations as follows:

$$\begin{align*}
\dot{e}_1 &= a(e_2 - e_1) - k_1 e_1 \\
\dot{e}_2 &= be_1 + e_4 - k e_2 - kxz' + kxz \\
\dot{e}_3 &= -ce_3 - ke_3 + |x'| - |x| \\
\dot{e}_4 &= -ge_1 - ke_4
\end{align*}$$  \hspace{1cm} (20)

According to Theorem 4.1, let $k > 0$. 8198 and the synchronization of the two hyperchaotic systems will be realized. In numerical simulations, the four-order Runge-Kutta method is used to solve the system (2). Choose the value of initial conditions of the driving the system (2): $x=2.2$, $y=2.2$, $z=0$, $w=0$, and of response system (19): $x'=1$, $y'=1$, $z'=1$, $w'=1$. Choose $k = 0.5, 1$, the wave form of the error variables and the portrait of corresponding variables are shown in Figs.4–5, respectively. From the figures, we can see that the synchronization error will converge into zero finally and two identical hyperchaotic systems with different initial values are indeed achieving chaos synchronization when $k > 0$. 8198 is satisfied.

### 5. Conclusions

In summary, in this paper, by changing the square term to be absolute term and state feedback control introduction in Liu chaotic system, a new 4-dimension hyperchaotic system is designed only containing one quadratic term. The hyperchaotic behavior is verified by Lyapunov exponents and Poincaré mapping. Based on Lyapunov second method, a control law is designed
to compel the hyperchaotic system to converge into zero equilibrium. The application of linear feedback control principle in this system shows that the control method can be used in low-dimension chaotic system and also can be used in high-dimension hyperchaotic system. It is essential to design a proper linear feedback control gain vector to use this synchronization method. It is convenient to construct a response system, and when linear feedback control gain is chosen to exceed the maximum Lyapunov exponent of the system, the synchronization gives good proof for the validity of this method.

Fig.4. $k = 0.5$. (a) The waveform of the error variables, (b) the phase portrait of corresponding state variables

Fig.5. $k = 1$. (a) The waveform of the error variables, (b) the phase portrait of corresponding state variables

References:


Forgery Attacks on Wang’s Signature Schemes Based on Factoring and Discrete Logarithm*

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Abstract: To enhance the security of signature schemes, Wang proposed two signature schemes based on the difficulties of simultaneously solving the factoring and discrete logarithm problems with almost the same sizes of arithmetic modulus. This paper, firstly, shows that Wang’s signature scheme 1 do not satisfy the claimed properties. Moreover, we will point out that the all variants of Wang’s scheme 2 are not secure if attackers can solve discrete logarithm problems.

Key words: cryptography; security; digital signature; factoring; discrete logarithm

1. Introduction

In 1976, Diffie and Hellman[1] proposed the concept of the public key cryptosystems. Many such cryptosystems[2,3], in which the security is based on a single cryptographic assumption, i.e., discrete logarithm or factoring a large composite number problem. Although schemes based on one of the above cryptographic assumptions appear secure today, they may be exploded in the future. As soon as this happens, cryptosystems based on such problem will no longer be secure. Hence, several cryptosystems based their security on solving multiple hard problems simultaneously were proposed. The major motivation is that it becomes very unlikely to solve the multiple hard problems simultaneously.

Based on this motivation, He[4] recently proposed a digital signature scheme based on the factoring and discrete logarithm problems and claimed that each user only requires one key pair in the public key cryptosystem. Hwang et al.[5] improved the efficiency of He’s scheme. Wang et al.[6] also proposed two signature scheme based on factoring and discrete logarithm.

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Pon, et al. [7], recently, investigated all eight variants of He’s digital signature scheme. Later, Shao Z. [8], Qian H. et al. [9] shown that the all eight variants of He’s digital signature scheme, as well as more variants, are not secure if attackers can solve discrete logarithm problems. In this paper, we will consider the security of the two signature schemes by Wang and point out that all of Wang’s signature schemes is based only on solving the discrete logarithm.

2. Brief Review of Wang’s Signature Schemes

There are three phases in these schemes: initialization, digital signature generation, and verification. A trust center selects the following parameters:

(1) a large prime \( P = 4p_1q_1 + 1, p_1 = 2p_2 + 1, q_1 = 2q_2 + 1, \) and \( R = p_1q_1, \) where \( p_1, p_2, q_1, q_2 \) are all large primes.

(2) an element \( g \) with order \( p_1q_1 \) in \( \mathbb{Z}_P. \)

In Wang’s schemes, each user selects a private key \( x \in \mathbb{Z}_R \) and obtains corresponding public key \( y \) as

\[
y = g^{x^2 + x^{-2}} \pmod{P}
\]

2.1. Wang’s Scheme 1

Each user selects a public integer \( e \) and computes the secret value \( d \) such that \( ed = 1 \pmod{\phi(R)}, \) where \( \gcd(e, R) = 1. \)

In the digital signature generation phase, suppose the signer wants to sign a message \( m. \) To generate signature for \( m, \) the signer performs the following steps.

1. Randomly selects an integer \( t \in \mathbb{Z}_R \) such that \( \gcd(t^2, R) = 1, \) and computes

\[
r = g^{t^2 + t^{-2}} \pmod{P}
\]

2. Then he evaluates \( s \)

\[
s = (s')^d \pmod{R}
\]

and \( s' \) is obtained by solving the equation (3),

\[
f(r, m) = (x + x^{-1})^2 r + (t + t^{-1})^2 s' \pmod{R}
\]

where \( f \) is a one-way hash function.

3. Sends \( (r, s) \) associated with \( m \) to the verifier.

The verifier computes the value:

\[
s' = (s)^e \pmod{R}
\]
Upon receiving the signature \((r, s)\) associated with \(m\) with respect to public key \(y\) of the signer and \(s'\), the verifier verifies the signature by checking the following congruent equality:

\[
g^{f(r,m)} = y^{r}r^{s}g^{2(r+s')}(mod\ P)
\]  

If the equation (6) holds, the \((r, s)\) is a valid signature.

### 2.2. Wang’s Scheme 2

In the digital signature generation phase, suppose the signer wants to sign a message \(m\). To generate signature for \(m\), the signer performs the following steps.

1. Randomly selects an integer \(t \in Z_{R}\) such that \(gcd(t^{2}, R) = 1\), and computes

\[
r = g^{x^2 + t^{-2}}(mod\ P)
\]  

2. Finds \(s\) satisfying

\[
f(r, m) = (x + x^{-1})^{2}r + (t + t^{-1})^{-2}s^{2}(mod\ R)
\]  

where \(f\) is a one-way hash function.

3. Sends \((r, s)\) associated with \(m\) to the verifier.

Upon receiving the signature \((r, s)\) associated with \(m\) with respect to public key \(y\) of the signer, the verifier verifies the signature by checking the following congruent equality:

\[
g^{f(r,m)} = y^{r}r^{s}g^{2(r+s')}(mod\ P)
\]  

If the equation (9) holds, the \((r, s)\) is a valid signature.

### 3. Security of Wang’s Signature Schemes

The symbols \(m, g, R, x, y, t\) and \(r\) are defined in the same way as in Section 2. All public keys and private keys are generated using the same conditions and the equations of Wang’s signature schemes. Three symbols set as follows:

\[
x' = x^2 + x^{-2}(mod\ R)
\]

\[
t' = t^2 + t^{-2}(mod\ R)
\]

\[
H = f(r, m)
\]

where \(f\) is a secure one-way hash function.

#### 3.1. Security of Wang’s Scheme 1

Firstly, we give a simple scheme, which is the same as Wang’s scheme intrinsically.

Each user selects a private key \(x' \in Z_{R}\) and obtains corresponding public key \(y\) as

\[
y = g^{x'}(mod\ P)
\]  

\[
H = f(r, m)
\]
Each user selects a public integer $e$ and computes the secret value $d$ such that $ed = 1 \pmod{\phi(R)}$, where $\gcd(e, R) = 1$.

In the digital signature generation phase, suppose the signer wants to sign a message $m$. To generate signature for $m$, the signer performs the following steps.

1. Randomly selects an integer $t' \in \mathbb{Z}_R$, and computes

$$ r = g^{t'} \pmod{P} $$

2. Then he evaluates $s$

$$ s = (s')^d \pmod{R} $$

and $s'$ is obtained by solving the following equation (13):

$$ f(r, m) = (x' + 2)r + (t' + 2)s' \pmod{R} $$

where $f$ is a one-way hash function.

3. Sends $(r, s)$ associated with $m$ to the verifier.

The verifier computes the value:

$$ s' = (s)^e \pmod{R} $$

Upon receiving the signature $(r, s)$ associated with $m$ with respect to public key $y$ of the signer and $s'$, the verifier verifies the signature by checking the following congruent equality:

$$ g^{f(r, m)} = y^{r + s'} g^{2(r + s')} \pmod{P} $$

If the equation (15) holds, the $(r, s)$ is a valid signature.

Let $(r, s)$ be the signature of message $m$. Assume that the discrete logarithm problem is solvable. We can obtain $t' \pmod{R}$ from $r$. From the equations (12) and (13), we have

$$ f(r, m) = (x' + 2)r + (t' + 2)s' \pmod{R} $$

From the equation (16), although there is one unknown variable $x'$ unfortunately, the variable is not updated from time to time. Therefore, the adversary can solve $x'$ after getting a valid message-signature pair.

3.2. Security of Wang’s Scheme 2

Similar to Pon S.F. et al.\cite{7}, in this section we will develop a complete list of 6 Wang-type digital signature schemes and analyze the security of the all Wang-type schemes.

The following forgery attack is under the assumption that attackers are able to solve the discrete logarithm problem, they can find an integer $z$ such that

$$ y = g^z \pmod{P} $$
Where, \( z = (x + x^{-1})^2 (\text{mod } R) \). Because \( \gcd((x + x^{-1})^2, R) = 1 \) so \( \gcd(z, R) = 1 \).

To forge a signature for any message \( m \), the attackers perform the following steps.

1. Randomly select an integer \( t \in \mathbb{Z}_R \) such that \( \gcd(t, R) = 1 \). Set \( t' = t^2 + t^{-2} \) and compute \( r = g^{t'} (\text{mod } P) \).
2. Compute \( H = f(r, m) \).
3. Find \( s \) satisfying \( a = b(z + 2) + c(t' + 2) (\text{mod } R) \) (12)

4. Sends the signature \((r, s)\) associated with \( m \) to the verifier.

Hence, the forged signature \((r, s)\) satisfies the verification equation (12).

where \((a, b, c)\) are three parameters taken from the three-element set \( \{H, r, s^2\} \), or formed by a mathematical combination of the three member elements \( H, r \) and \( s^2 \).

Ignoring the difference between the signed symbols + and − in the equation(12), all the possible variants of the Wang’s digital schemes listed in Table 1.

In first signature scheme, the equation(12) is

\[ H = r(x + 2) + s^2(t' + 2)(\text{mod } R) \]

In 2nd signature scheme, the equation(12) is

\[ H = s^2(x + 2) + r(t' + 2)(\text{mod } R) \]

In 3rd signature scheme, the equation(12) is

\[ s^2 = H(x + 2) + r(t' + 2)(\text{mod } R) \]

In 4th signature scheme, the equation(12) is

\[ s^2 = r(x + 2) + H(t' + 2)(\text{mod } R) \]

In 5th signature scheme, the equation(12) is

\[ r = s^2(x + 2) + H(t' + 2)(\text{mod } R) \]

In 6th signature scheme, the equation(12) is

\[ r = H(x + 2) + s^2(t' + 2)(\text{mod } R) \]

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
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<tbody>
<tr>
<td><strong>Signature equation (mod R)</strong></td>
</tr>
<tr>
<td>(1) ( H = r(z + 2) + s^2(t' + 2) )</td>
</tr>
<tr>
<td>(2) ( H = s^2(z + 2) + r(t' + 2) )</td>
</tr>
<tr>
<td>(3) ( s^2 = H(z + 2) + r(t' + 2) )</td>
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<tr>
<td>(4) ( s^2 = r(z + 2) + H(t' + 2) )</td>
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<tr>
<td>(5) ( r = s^2(z + 2) + H(t' + 2) )</td>
</tr>
<tr>
<td>(6) ( r = H(z + 2) + s^2(t' + 2) )</td>
</tr>
</tbody>
</table>
Hence, all the five variants of the Wang’s digital signatures, as well as the Wang’s scheme 2, are not secure if attackers can solve discrete logarithm problems.

In fact, for Wang’s scheme 2, we have that

\[ g^H = g^{r(x+2)}g^{s^2(t+2)} = (g^x)^r g^{2r} g^{s^2t^2} g^{2s^2} = y^r s^{2r} g^{2(r+s^2)} \]

By using a similar way, the attacks can forge a signature scheme of 2—6th signature schemes for any message \( m \).

Therefore, these variants of the Wang’s signature scheme are at most based on the difficulties of solving the discrete logarithm problems.

4. Conclusion

We have presented a generalized forgery attack against the Wang’s signature schemes based on factoring and discrete logarithms. If the attackers can solve the discrete logarithm problem, they can easily forgery signature schemes for two schemes by Wang and another Wang-type schemes. Therefore, the Wang’s schemes are not secure as the author claimed and are not based on two cryptographic assumptions simultaneously. How to design efficient digital signature schemes based on two cryptographic assumptions simultaneously still remains an open problem.

References:

The Edge-pancyclicity of Generalized Hypercubes*

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Abstract: Generalized hypercubes (denoted by $Q(d_1, d_2, \cdots, d_n)$) is an important network topology for parallel processing computer systems. Some methods of forming big cycle from small cycles and links have been developed. Basing on which, we has proved that in generalized hypercubes, every edge can be contained on a cycle of every length from 3 to $|V(G)|$ inclusive and all kinds of length cycles have been constructed. The edge-panciclicity and node-pancilicity of generalized hypercubes can be applied in the topology design of computer networks to improve the network performance.

Key words: complete graph; pancyclicity; generalized hypercube

1. Introduction

It is well known that interconnection networks play an important role in parallel computing/communication systems. One of the central issues in evaluating a network is to study the embedding problem. When a network is modeled by a graph, the embedding problem asks if a guest graph is a subgraph of a host graph, and an important benefit of graph embedding is that we can apply existing algorithms for guest graphs to host graphs. This problem has attracted a burst of studies in recent years. Cycle networks are suitable for designing simple algorithms with low communication cost. Since some parallel applications, such as those in image and signal processing, are originally designated on a cycle architecture, it is important to have effective cycle embedding in a network. A graph is pancyclic if it contains cycles of every length from its girth to order inclusive. A graph is of pancyclicity if it is pancyclic. The pancyclicity and cycle-embedding of many networks has been investigated in [1-4].

The structural character of Generalized Hypercubes($Q(d_1d_2\cdots d_n)$) and its special form $Q_n(m)$ (when $d_1 = d_2 = \cdots = d_n = m$) have attracted many attentions in [1, 5–10] and [11–13]. Paper [7] mainly focused on developing the routing algorithm of generalized hypercubes.

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The fault-tolerant routing algorithm of generalized hypercubes also have obtained substantial discussion [11–13]. The wide diameter and fault diameter of generalized hypercube networks have been offered in [10]. Paper [8, 9] showed the pancyclicity and edge-pancyclicity of $m$-ary $n$-cube networks.

Efficient communication is critical to the performance of a generalized hypercube system. The reliability of systems, which guarantees efficient transmission of messages, often depends on the structure of network. Containing more cycles in a network can promote the connectivity, reliability, and efficiency greatly. Cycle structure is an important issue that need to be studied extensively. In fact cycle structure is an elementary topology structure for parallel and distribute computing, and has vast applications in the local area network as well as the parallel computing algorithms. It is also used as a control structure of the data flow of distributed computing. Because of its good nature, cycle structure constantly attracts researchers from areas of computing and graph theory.

If a graph contains all cycles with lengths from 3 to $|V(G)|$, then it is called Pancyclic. Furthermore, if every edge in a graph $G$ can be contained on a cycle of every length from 3 to $|V(G)|$, then the graph is called Edge-Pancyclic. In this paper, the structure property of the generalized hypercube network topology is analyzed, based on which some good properties have been revealed and used to prove the fact that $Q(d_1d_2\cdots d_n)$ is an excellent topological structure of computer networks with guaranteed reliability and efficiency in data transmission. The edge-pancyclicity can be used to improve the topology design of computer networks. Furthermore, some methods of forming big cycle from small cycles and links have been developed.

The rest of this paper is organized as follows. Section 2 gives a basic definition and some lemmas used in our discussion. The proofs of our main results are in Section 3. Some conclusions are given in Section 4.

2. Preliminaries

Suppose $G(V, E)$ is a simple graph. The set of vertices and the set of edges of $G$ are denoted by $V(G) = V$, and $E(G) = E$, respectively. If $xy \in E$, then $x$ and $y$ are neighbor vertices. $G$ is $k$-regular if the degrees of all vertices is $k$. The path $P(x, y)$ connecting two vertices $x$ and $y$ is an ordered vertices sequence with start vertex $x$ and end vertex $y$. If the start vertex of a path is the same as the end vertex, then the path is a cycle. The length of path $P(x, y)$ is the number of edges connecting the ordered vertices sequence in the path.

**Definition 1** Generalized hypercubes is a graph $Q(d_1d_2\cdots d_n)$. The set of vertices is denoted as $V = \{x_1x_2\cdots x_n : x_i \in \{0, 1, \cdots, d_i - 1\}, i = 1, 2, \cdots, n\}$. This means that every vertex can be represented by an $n$-bit string, in which each bit can take one of $d_i$ possible values. Two vertices $x$ and $y$ are adjacent if and only if $x$ differs from $y$ by exactly one bit, i.e., if $x = (x_1x_2\cdots x_n)$, and $y = (y_1y_2\cdots y_n)$, then $xy \in E$ if and only if there is exactly one $i$ such
that \( x_i \neq y_i \) but \( x_j = y_j \) for all \( 1 \leq i \neq j \leq n \).

From Definition 1, \( Q(d_1d_2 \cdots d_n) \) contains \( d_1d_2 \cdots d_n \) vertices and \( |d_1d_2 \cdots d_n(d_1 + d_2 + \cdots + d_n - n)|/2 \) edges. If \( n = 1 \), it is a complete graph with \( d_1 \) nodes, that is, there exists a link between any two nodes. If \( d_1 = d_2 = \cdots = d_n = 2 \), it is the well-known \( n \) dimensional hypercube. From the structure of \( Q(d_1d_2 \cdots d_n) \) it is easy to derive the following two properties.

**Lemma 1** \( Q(d_1d_2 \cdots d_n) \) is \((d_1 + d_2 + \cdots + d_n - n)- regular with diameter \( n \) and the connectivity is \( d_1 + d_2 + \cdots + d_n - n \).

**Lemma 2** \( Q(d_1d_2 \cdots d_n) \) can be decomposed into \( d_i \) subsystems with respect of any component, i.e., \( (n-1) \)-cube systems \( Q^i(d_1 \cdots d_i, d_{i+1} \cdots d_n), 0 \leq i \leq d_i - 1 \). For every vertex \( u \in V(Q^i(d_1 \cdots d_i, d_{i+1} \cdots d_n)), 0 \leq i \leq d_i - 1 \), vertex \( u \) has and only has \( d_i - 1 \) neighbors not in \( Q^i(d_1 \cdots d_i, d_{i+1} \cdots d_n) \). These \( d_i - 1 \) neighbors are contained in \( d_i - 1 \), \( (n-1) \)-cube \( Q^j(d_1 \cdots d_i, d_{i+1} \cdots d_n), 0 \leq j \neq i \leq d_i - 1 \), but all of the rest neighbors of \( u \) are contained in \( V(Q^j(d_1 \cdots d_i, d_{i+1} \cdots d_n)) \).

### 3. Main Results

**Lemma 3** Suppose \( x_1x_2 \cdots x_nx_1 \) is a cycle of graph \( G \). If there exists a \( u \in V(G) - \{x_1, x_2, \cdots, x_n\} \) and \( ux_i, ux_{i+1} \in E(G) \), then \( x_1x_2 \cdots x_ix_{i+1} \cdots x_nx_1 \) is also a cycle of length \( n+1 \) in \( G \).

**Lemma 4** Suppose that \( C = x_1x_2 \cdots x_nx_1 \) is a cycle of length \( n \) in \( G \). If there exists an edge \( y_iy_{i+1} \in E(G) - E(C) \) and \( x_iy_i, x_{i+1}y_{i+1} \in E(G) \), then \( C + x_iy_j + x_{i+1}y_{i+1} - x_iy_{i+1} \) forms a new cycle of length \( n+2 \) in \( G \).

**Lemma 5** Suppose that \( C_1 = x_1x_2 \cdots x_nx_1 \), and \( C_2 = y_1y_2 \cdots y_ky_1 \) are two disjoint cycles of graph \( G \). If there exist \( i \) and \( j \) such that \( x_iy_j, x_{i+1}y_{j+1} \in E(G) \) then \( C_1 \cup C_2 - x_iy_1 - y_jy_{j+1} + x_{i+1}y_1 + x_1y_{j+1} \) forms a new cycle of length \( n+k \) in \( G \).

**Lemma 6** In \( Q(d_1d_2) \), every edge is contained in all cycles of length \( l, 3 \leq l \leq d_1d_2 \). Namely \( Q(d_1d_2) \) is a edge-pancyclic graph.

**Proof** Decompose \( Q(d_1d_2) \) into \( d_1 \) complete subgraphs with respect to the first bit to obtain \( Q^i(id_2), 0 \leq i \leq d_1 - 1 \).

**Case 1** Choose any edge \( x^iy^i \in E(Q^i(id_2)), i = 0, 1, \cdots, d_1 - 1 \). Let \( x^i = ib_2 \), and \( y^i = ic_2 \in V(Q^i(id_2)) \). We will show that the edge \( x^iy^i \) is contained in the cycles of length \( l, 3 \leq l \leq d_1d_2 \).

**Case 1.1** If \( 3 \leq l \leq d_2 \), the edge \( x^iy^i \) is contained in a cycle of length \( l \) in any \( Q^i(id_2) \).

**Case 1.2** If \( l = d_2 + 1 \), choose a cycle \( C \) of length \( d_2 - 1 \) in \( Q^i(id_2) \), which contains the edge \( x^iy^i \). Take an edge \( (ju_2, kv_2) \) in any other \( Q^j(id_2) \) such that the corresponding edge \( (iu_2, iv_2) \) is on the cycle \( C \) \((b_2 \neq u_2 \text{ and } c_2 \neq v_2, \text{i.e., the edge formed by the pair of the neighbors of the end vertex of the selected edge in } Q^j(id_2) \text{ is also on the cycle } C)\). From Lemma 3 a cycle of length \( d_2 + 1 \) can be obtained, which contains the edge \( x^iy^i \).
Case 1.3 If \( l = d_2 + 2 \), similar to Case 1.2, choose a cycle \( C \) of length \( d_2 \) in \( Q^i(id_2) \), which contains the edge \( x^iy^i \). Take an edge \((ju_2,jv_2)\) in any other \( Q^j(jd_2) \) such that the corresponding edge \((iu_2,iv_2)\) is on the cycle \( C \), \( b_2 \neq u_2 \) and \( c_2 \neq v_2 \). By Lemma 4 a cycle of length \( d_2 + 2 \) can be obtained, which contains the edge \( x^iy^i \).

Case 1.4 If \( l = d_2 + r, 3 \leq r \leq d_2 \), choose a cycle \( C \) of length \( d_2 \) in \( Q^i(id_2) \), which contains the edge \( x^iy^i \). Take a cycle of length \( r \) in any other \( Q^j(jd_2) \) such that the corresponding edge (but not \( x^iy^i \)) is on the cycle \( C \). By the same construction method of Lemma 5 a cycle of length \( d_2 + r \) can be obtained, which contains the edge \( x^iy^i \).

Case 1.5 If \( l = kd_2 + 1, 2 \leq k \leq d_1 - 1 \), choose \( k \) cycles \( C_1, C_{i_1}, C_{i_2}, \ldots, C_{i_{k-1}} \) of length \( d_2 \) in \( Q^i(id_2) \), \( Q^{i_1}(i_1d_2) \), \( Q^{i_2}(i_2d_2) \), \ldots, \( Q^{i_{k-1}}(i_{k-1}d_2) \), respectively, such that two neighbor cycles have at least one corresponding edge (but not \( x^iy^i \)) on the neighbor cycle. Especially, \( C_1 \) contains the edge \( x^iy^i \). Since all \( Q^i(id_2) \) are complete graph, it is feasible. Also take a node in \( Q^{i_1}(i_1d_2) \). According to Lemma 5 and the method of Lemma 3, a cycle of length \( kd_2 + 1 \) can be constructed, which contains the edge \( x^iy^i \).

Case 1.6 If \( l = kd_2 + 2, 2 \leq k \leq d_1 - 1 \), choose \( k \) cycles \( C_1, C_{i_1}, C_{i_2}, \ldots, C_{i_{k-1}} \) of length \( d_2 \) in \( Q^i(id_2) \), \( Q^{i_1}(i_1d_2) \), \( Q^{i_2}(i_2d_2) \), \ldots, \( Q^{i_{k-1}}(i_{k-1}d_2) \), respectively, such that two neighbor cycles have at least one corresponding edge (but not \( x^iy^i \)) on the neighbor cycle. Especially, \( C_1 \) contains the edge \( x^iy^i \). Since all \( Q^i(id_2) \) are complete graph, it is feasible. Also take two nodes in \( Q^{i_1}(i_1d_2) \). According to Lemma 4 and Lemma 5, a cycle of length \( kd_2 + 2 \) can be constructed, which contains the edge \( x^iy^i \).

Case 1.7 If \( l = kd_2 + r, 2 \leq k \leq d_1 - 1, 3 \leq r \leq d_2 \), choose \( k \) cycles \( C_1, C_{i_1}, C_{i_2}, \ldots, C_{i_{k-1}} \) of length \( d_2 \) in \( Q^i(id_2) \), \( Q^{i_1}(i_1d_2) \), \( Q^{i_2}(i_2d_2) \), \ldots, \( Q^{i_{k-1}}(i_{k-1}d_2) \), respectively. Especially, \( C_1 \) contains the edge \( x^iy^i \). Also take \( r \) nodes in \( Q^{i_1}(i_1d_2) \) to form a cycle \( C \) such that the bigger cycle formed by \( \{C,C_1,C_{i_1},C_{i_2},\ldots,C_{i_{k-1}}\} \) satisfies the condition that any two neighbor cycles have at least a pair of corresponding edges (but not \( x^iy^i \)) on the cycle.

According to Lemma 5, a cycle of length \( kd_2 + r \) can be constructed, which contains the edge \( x^iy^i \).

Case 2 Choose any edge \( x^ix^j \in E(Q(d_1d_2)), i \neq j \in \{0,1,\ldots,d_1-1\} \), Suppose \( x^i = ib_2 \in V(Q^i) \), and \( x^j = jb_2 \in V(Q^j) \). Then by Lemma 2 also can think the node \( x^i \) and \( x^j \) are contained in the same subgraph if \( Q(d_1d_2) \) can be decomposed into \( d_2 \) subgraphs with respect of the second bit. So the edge \( x^ix^j \) is contained in the cycles of length \( l, 3 \leq l \leq d_1d_2 \) similar to Case 1.

Theorem 1 \( Q(d_1d_2 \cdots d_n) \) is a edge-pancyclic graph.

Proof The theorem is proved by induction.

When \( n = 1 \), \( Q(d_1) \) is a \( d_1 \)-complete graph, and therefore is a edge-pancyclic graph.

When \( n = 2 \), according to Lemma 6, \( Q(d_1d_2) \) is a edge-pancyclic graph.

Suppose \( Q(d_1d_2 \cdots d_k) \) is edge-pancyclic for \( k < n \). In the following consider \( k = n \).

Decompose \( Q(d_1d_2 \cdots d_n) \) into \( d_1 \) subsystems with respect of the first bit, \((n-1)-\)cube
$Q^0(0d_2 \cdots d_n), Q^1(1d_2 \cdots d_n), \ldots, Q^{d_1-1}(d_1 - 1d_2 \cdots d_n)$. By the induction hypothesis, $Q^i(id_2 \cdots d_n)$, $0 \leq i \leq d_1 - 1$ are edge-pancyclic.

**Case 1** Choose any edge $x^iy^i \in E(Q^i(id_2 \cdots d_n))$, $i = 0, 1, \ldots, d_1 - 1$. Suppose $x^i = ia_2a_3 \cdots a_n$, and $y^i = ib_2b_3 \cdots b_n \in V(Q^i)$. We will show that the edge $x^iy^i$ is contained in the cycles of length $l$, $3 \leq l \leq d_1d_2 \cdots d_n$.

If $3 \leq l \leq d_2 \cdots d_n$, a cycle of length $l$ which contains the edge $x^iy^i$ can be obtained in any $Q^i(id_2 \cdots d_n), 0 \leq i \leq d_1 - 1$. Therefore we need only to consider the situation of $l \geq d_2 \cdots d_n + 1$, which can be divided into the following cases.

**Case 1.1** $l = d_2 \cdots d_n + 1$.

Select a Hamilton cycle $C$ with length $l - 2 = d_2 \cdots d_n - 1$ in some $Q^i(id_2 \cdots d_n)$, which contains the edge $x^iy^i$. Choose an edge $x^iz^i \in E(Q^i(id_2 \cdots d_n))$. Suppose $z^i = ic_2c_3 \cdots c_n \in V(Q^i(id_2 \cdots d_n))$. In another $Q^j(jd_2 \cdots d_n), 0 \leq j \neq i \leq d_1 - 1$, find its corresponding edge $x^jz^j \in E(Q^j(jd_2 \cdots d_n))$ so that $x^j = ja_2a_3 \cdots a_n$, and $z^j = jc_2c_3 \cdots c_n \in V(Q^j(jd_2 \cdots d_n))$.

By Lemma 4 a cycle $C - x^iz^i + x^jz^j + x^ix^j + z^iz^j$ of length $l$ can be constructed, which contains the edge $x^iy^i$.

**Case 1.2** $l = d_2 \cdots d_n + 2$.

Select a Hamilton cycle $C$ with length $l - 2 = d_2 \cdots d_n - 1$ in some $Q^i(id_2 \cdots d_n)$, which contains the edge $x^iy^i$. Choose an edge $x^iz^i \in E(Q^i(id_2 \cdots d_n))$. Suppose $z^i = ic_2c_3 \cdots c_n \in V(Q^i(id_2 \cdots d_n))$. In another $Q^j(jd_2 \cdots d_n), 0 \leq j \neq i \leq d_1 - 1$, choose its corresponding edge $x^jz^j \in E(Q^j(jd_2 \cdots d_n))$ so that $x^j = ja_2a_3 \cdots a_n$, and $z^j = jc_2c_3 \cdots c_n \in V(Q^j(jd_2 \cdots d_n))$.

By Lemma 4 a cycle $C - x^iz^i + x^jz^j + x^ix^j + z^iz^j$ of length $l$ can be constructed, which contains the edge $x^iy^i$.

**Case 1.3** $l = d_2 \cdots d_n + r, 3 \leq r \leq d_2 \cdots d_n$.

Select a Hamilton cycle $C$ with length $d_2 \cdots d_n$ in some $Q^i(id_2 \cdots d_n)$, which contains the edge $x^iy^i$. Choose an edge $x^iz^i \in E(Q^i(id_2 \cdots d_n))$. Suppose $z^i = ic_2c_3 \cdots c_n \in V(Q^i(id_2 \cdots d_n))$. In another $Q^j(jd_2 \cdots d_n), 0 \leq j \neq i \leq d_1 - 1$, find its corresponding edge $x^jz^j \in E(Q^j(jd_2 \cdots d_n))$ so that $x^j = ja_2a_3 \cdots a_n$, and $z^j = jc_2c_3 \cdots c_n \in V(Q^j(jd_2 \cdots d_n))$. Since $Q^j(jd_2 \cdots d_n)$ is symmetric and edge-pancyclic, a cycle $C_1$ with length $r$ containing edge $x^jz^j \in E(Q^j_{m-1}(m))$. By Lemma 5 a cycle $C - x^iz^i - x^jz^j + x^ix^j + z^iz^j + C_1$ of length $l$ can be constructed, which contains the edge $x^iy^i$.

**Case 1.4** $l = kd_2 \cdots d_n + 1, 2 \leq k \leq d_1 - 1$.

According to Case 1.1, a cycle $C$ of length $d_2 \cdots d_n + 1$ containing the edge $x^iy^i$ can be constructed from $d_2 \cdots d_n - 1$ vertices of $Q^i(id_2 \cdots d_n)$ and two vertices of $Q^{i_0}(i_0d_2 \cdots d_n)$. Select a Hamilton cycle $C_{i_1}$ in $Q^{i_1}(i_1d_2 \cdots d_n)$ such that there are at least two edges of $C_{i_1}$ whose corresponding two edges of $Q^i(id_2 \cdots d_n)$ are on the cycle $C$. Since $Q^{i_1}(i_1d_2 \cdots d_n)$ is symmetric, this is feasible. Using the same method as Lemma 4, choose a cycle of length $2d_2 \cdots d_n + 1$ formed by a pair of corresponding edges of cycle $C_{i_1}$ and $C$. $k-2$ Hamilton cycles of order $d_2 \cdots d_n$, $C_{i_2}, C_{i_3}, \ldots, C_{i_{k-1}}$ can be obtained from $Q^{i_2}(i_2d_2 \cdots d_n), Q^{i_3}(i_3d_2 \cdots d_n), \ldots, Q^{i_{k-2}}$.
By Property 2, the end vertices of the corresponding edges of $C_{i_k}, C_{i_{h}} (2 \leq g \neq h \leq d_1 - 1)$, are neighbors respectively. Therefore from Lemma 4, $l = d_2 \cdots d_n + 1, 2 \leq k \leq d_1 - 1$, which contains the edge $x^i y^j$.

**Case 1.5** $l = k d_2 \cdots d_n + 2, 2 \leq k \leq d_1 - 1$.

Similar to Case 1.4, $k$ Hamilton cycles of length $d_2 \cdots d_n$. $C_i, C_{i_1}, C_{i_2}, \cdots, C_{i_{k-1}}$ can be obtained from $Q^i (i d_2 \cdots d_n), Q^{i_1} (i_1 d_2 \cdots d_n), Q^{i_2} (i_2 d_2 \cdots d_n), \cdots, Q^{i_{k-1}} (i_{k-1} d_2 \cdots d_n)$, respectively, according to the orders of the vertices. Choose an edge $x^i y^k \in E (Q^i (i d_2 \cdots d_n))$ such that the corresponding edge $x^{k-1} y^{k-1} \in E (Q^{i_{k-1}} (i_{k-1} d_2 \cdots d_n))$ is on $C_{i_{k-1}}$. According to Lemma 5, a cycle of length $l = k d_2 \cdots d_n + 2, 2 \leq k \leq d_1 - 1$, which contains the edge $x^i y^j$.

**Case 1.6** $l = k d_2 \cdots d_n + r, 2 \leq k \leq d_1 - 1, 3 \leq r \leq d_2 \cdots d_n$.

Similar to Case 1.5, $k$ Hamilton cycles of length $d_2 \cdots d_n$. $C_i, C_{i_1}, C_{i_2}, \cdots, C_{i_{k-1}}$ can be obtained from $Q^i (i d_2 \cdots d_n), Q^{i_1} (i_1 d_2 \cdots d_n), Q^{i_2} (i_2 d_2 \cdots d_n), \cdots, Q^{i_{k-1}} (i_{k-1} d_2 \cdots d_n)$, respectively, according to the orders of the vertices. Choose a cycle $C$ of length $r$ from $Q^{i_k} (i_k d_2 \cdots d_n)$ such that there is at least one corresponding edge is on $C_{i_{k-1}}$. Since $Q^{i_k} (i_k d_2 \cdots d_n)$ is symmetric, it is feasible. According to Lemma 5, $C_i, C_{i_1}, C_{i_2}, \cdots, C_{i_{k-1}}, C_{i_k}$ form a cycle of length $l = k d_2 \cdots d_n + r, 2 \leq k \leq d_1 - 1, 3 \leq r \leq d_2 \cdots d_n$, which contains the edge $x^i y^j$.

Case 2 choose any edge $x^i x^j \in E (Q (d_1 d_2 \cdots d_n)), i \neq j \in \{0, 1, \cdots, d_1 - 1\}$, Suppose $x^i = i a_2 a_3 \cdots a_n \in V (Q^i)$, and $x^j = j a_2 a_3 \cdots a_n \in V (Q^j)$. Then by Lemma 2 we also can think the node $x^i$ and $x^j$ are contained in the same subgraph if $Q (d_1 d_2 \cdots d_n)$ can be decomposed into $d_2$ subgraphs with respect of the second bit. So the edge $x^i x^j$ is contained in the cycles of length $3 \leq l \leq d_1 d_2 \cdots d_n$ similar to Case 1.

**Corollary 1** $Q (d_1 d_2 \cdots d_n)$ is a node-pancyclic graph.

**Proof** This is evident by Theorem 1.

### 4. Conclusion

Network topology is an important issue in the design of computer networks since it is crucial to many different choices, generalized hypercube is an important network topology for parallel processing computer systems. The Hamilton property and the cycle structure of the topology graph are important properties determining the performance of the network. This paper has performed the topological structure analysis of methods of forming big cycle have been developed, by using which every edge is contained on all cycles with length from 3 to $|V (Q (d_1 d_2 \cdots d_n))|$, which then has proved that the generalized hypercube is a pancyclic graph. This Hamilton property of generalized hypercube implies the reliability of $Q (d_1 d_2 \cdots d_n)$ and there exists many data transmitting routs. This can be used to improve the topology design of computer networks.
References:


Impact of Different Forms of Interest Rate Differential on the Flexible Price Monetary Model

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Abstract: In this paper, in order to investigate whether the impact of different forms of interest rate differential may pass on to the flexible price monetary model, two flexible price monetary models, which are separately derived from the generalized monetary models with log-level interest rate differential and that with interest rate differential, are tested for China yuan to US dollar exchange rate. Through Johansen maximum likelihood method, we find that there is little support in the cointegrating coefficient estimates for both flexible price monetary models for yuan/dollar exchange rate. However, the latter is generally better than the former in the light of sum of squared residual and log likelihood statistics. Therefore, we conclude that there is no transitive impact of different forms of interest rate differential on the flexible price monetary model.

Key words: flexible price monetary model; form of interest rate differential; cointegration test

1. Introduction

Monetary theory of exchange rate determination is the foundation stone of the development of modern theory of exchange rate determination. It has far-reaching meanings in terms of theory and political instruction for its rigorous logical relation between exchange rate and fundamentals in the model. But testing results change with different sample periods, different countries and different methods. Up to now, there is still no consistent view on the monetary model of exchange rate determination[1].

In 1970s, monetary models could explain fluctuations in exchange rates very well (Frankel, 1979); but after the 1980s, large numbers of empirical test results showed that monetary models explained fluctuations in exchange rates badly (Meese and Rose,1983)[2]. Since 1990s,
many researchers began to amend monetary models in terms of data choosing and testing methods\cite{3,4}. Mark\cite{1995} using the non-parameter regression equation concluded that monetary models could forecast exchange rate correctly\cite{5}; however, Berkowitz and Giorgianni\cite{2001} got the opposite conclusion after amending the data generate process flaw in Mark\cite{1995}\cite{6}; Mark and Sul\cite{2001} and Papach and Wohar\cite{2004} found considerable support for the monetary model using panel procedures\cite{7}, but considering the homogeneity restrictions, if the data generate process is different across different countries, then the wrong data panel procedure will get wrong conclusion\cite{8,9}.

By testing generalized monetary model for Japanese yen to US dollar and German marks to US dollar exchange rate during 1980–1997, Guoling Pan \cite{2000} found that testing results of the model with log-level interest rate differential were better than those of the model with interest rate differential generally\cite{10}. Because many modern exchange rate theories (like flexible, sticky price monetary model, theory of portfolio balance exchange rate etc) are based on the generalized monetary model, we would like to see if this change of form of interest rate differential would also have impact on the testing results of other models.

In this paper, in order to investigate whether there is transitive impact of different forms of interest rate, two flexible price monetary models, which are separately derived from the generalized monetary model with log-level interest rate differential and that with interest rate differential, was tested for yuan/dollar exchange rate. We first use Johansen maximum likelihood method to test cointegrating relationship between nominal exchange rate and fundamentals, and investigate whether there is any support in the cointegrating coefficient estimates for the flexible price monetary models for yuan/dollar exchange rate. Then we compare and analyze the empirical results of the two models, to see whether the impact of different forms of interest rate differential may pass on to the flexible price monetary models. Finally, we try to explore reasons for the flexible price monetary model unfit for yuan/dollar exchange rate.

2. Flexible Price Monetary Model

The monetary model explains exchange rate determination in terms of the demand for and the supply of money. The early, flexible-price monetary model built the theory of exchange rate determination based upon two critical assumptions: continuous Purchasing Power Parity(PPP) and stable money demand for both home and foreign countries\cite{11}.

Generalized currency quantity equation is

\[ \frac{M}{P} = KY^{a}i^{-b} \]

(1)

Where \( M \) is monetary supply, \( P \) is price level, \( Y \) is real income, \( i \) is interest rate, \( K \) is a constant, \( a \) is the income elasticity of demand for real money balances, and \( b \) is the interest rate semi-elasticity of real money demand.

Foreign countries have the same general currency quantity equation, asterisks denote foreign
variables, that is,
\[
\frac{M}{P} = K^* Y^* a_i^{* - b} \tag{2}
\]

The foreign and domestic elasticities are assumed to be identical for simplicity.

Given that absolute purchasing power parity holds, the nominal exchange rate \((\frac{P}{P^*})\) is equal to the domestic-foreign price ratio \((\frac{P}{P^*})\). Substituting the equations (1) and (2) into PPP, we get
\[
S = \frac{P}{P^*} = \frac{K^*}{K} \cdot \frac{M}{M^*} \cdot \left(\frac{Y}{Y^*}\right)^a \cdot \left(\frac{i}{i^*}\right)^{-b} \tag{3}
\]

Taking logarithms of the above equation and adding time factor \(t\), we obtain
\[
s_t = -(k - k^*) + (m_t - m_t^*) - a(y_t - y_t^*) + b(\ln i_t - \ln i_t^*) \tag{4}
\]

This is the generalized monetary model. Where a lower-case letter denotes the logarithms of the variable. To differ from the original-level of interest rate \(i\), we use \(\ln i\) to denote the log-level of interest rate.

If we further assume that Uncovered Interest Parity (UIP) holds or that the foreign exchange market is in equilibrium, then in a risk neutral world the interest rate differential \((i_t - i_t^*)\) must equal the expected percentage change of the nominal exchange rate. That is,
\[
i_t - i_t^* = E_t s_{t+1} - s_t \tag{5}
\]

Where \(E_t s_{t+1}\) is the expected \(t + 1\) spot exchange rate at time \(t\).

To make the monetary model fit for short term analysis of exchange rate, the equation (5) need to be introduced into the monetary model. So change the form of the generalized currency quantity equation, that is,
\[
\frac{M}{P} = K^* Y^* e^{-b i} \tag{6}
\]

Then we can get another form of the generalized monetary model, that is,
\[
s_t = -(k - k^*) + (m_t - m_t^*) - a(y_t - y_t^*) + b(i_t - i_t^*) \tag{7}
\]

Substituting the equation (5) into the equation (7), we get,
\[
s_t = -(k - k^*) + (m_t - m_t^*) - a(y_t - y_t^*) + b(E_t s_{t+1} - s_t) \tag{8}
\]

Under relative PPP, that is,
\[
S = \varphi \cdot \frac{P}{P^*} \tag{9}
\]

Where \(\varphi\) differs from one.

Taking logarithms and expressing the above equation in changes,
\[
\Delta s_{t+1} = \Delta p_{t+1} - \Delta p_{t+1}^* = \pi_{t+1} - \pi_{t+1}^* \tag{10}
\]

Where \(\pi_t\) and \(\pi_t^*\) are the domestic and foreign inflation rates at time \(t + 1\), respectively. Taking the expected form of the equation (10), that is,
\[
E_t s_{t+1} - s_t = E_t (\pi_{t+1} - \pi_{t+1}^*) \tag{11}
\]
The equation (11) states that the expected percentage changes of the nominal exchange rate equals the expected inflation differential between home and foreign countries. If we substituting this relationship into the equation (8), we get the flexible price monetary model. That is,
\[ s_t = -(k - k^*) + (m_t - m_t^*) - a(y_t - y_t^*) + bE_t(\pi_{t+1} - \pi_{t+1}^*) \] (12)

Guoling Pan (2000) found that testing results of the model (4) were generally better than the model (7) for Japanese yen to US dollar and German marks to US dollar exchange rate during 1980–1997. And the only difference between the two models lies in different forms of interest rate differential. So in order to investigate whether the impact of different forms of interest rate differential can pass on to the flexible price monetary model, which is derived from the generalized monetary model, we change the form of the equation (5), that is,
\[ i_t = i_t^* + E_t s_{t+1} - s_t \] (13)

Taking logarithms of the above equation, we obtain
\[ \ln i_t = \ln[i_t^* + (E_t s_{t+1} - s_t)] \] (14)

Substituting the equation (14) into the equation (4), we get
\[ s_t = -(k - k^*) + (m_t - m_t^*) - a(y_t - y_t^*) + b\{\ln[i_t^* + (E_t s_{t+1} - s_t)] - \ln i_t^*\} \] (15)

Then substituting the equation (11) into the equation (15), we get the other form of flexible price monetary model. That is,
\[ s_t = -(k - k^*) + (m_t - m_t^*) - a(y_t - y_t^*) + b\{\ln[\pi_{t+1}^* + E_t(\pi_{t+1} - \pi_{t+1}^*]) - \ln i_t^*\} \] (16)

The difference between the equation (16) and the equation (12) lies in their last items. \{\ln[i_t^* + E_t(\pi_{t+1} - \pi_{t+1}^*]) - \ln i_t^*\} minus \( E_t(\pi_{t+1} - \pi_{t+1}^*]) \) is denoted by \( D \), that is,
\[ D = \{\ln[i_t^* + E_t(\pi_{t+1} - \pi_{t+1}^*]) - \ln i_t^*\} - E_t(\pi_{t+1} - \pi_{t+1}^*]) \]
\[ = \ln \left[ \frac{i_t^* + E_t(\pi_{t+1} - \pi_{t+1}^*])}{i_t^*} \right] - \ln e^{E_t(\pi_{t+1} - \pi_{t+1}^*])} \] (17)

According to Taylor expansion \( e^x \approx 1 + x \),
\[ D = \ln \left[ 1 + \frac{E_t(\pi_{t+1} - \pi_{t+1}^*])}{i_t^*} \right] - \ln[1 + E_t(\pi_{t+1} - \pi_{t+1}^*])] \] (18)

So the difference between the equation (16) and the equation (12) actually depends on the difference between expected inflation rate differential adjusted by foreign interest rate \( \left( E_t(\pi_{t+1} - \pi_{t+1}^*]) \right) \) and expected inflation rate differential \( E_t(\pi_{t+1} - \pi_{t+1}^*]) \).

In next section, we will do empirical study on the two flexible price monetary models. To simplify, we use \( R \) to denote \{\ln[i_t^* + E_t(\pi_{t+1} - \pi_{t+1}^*]) - \ln i_t^*\} in the following section.

3. Data and Empirical Study

3.1. Data
Data set in this paper contains quarterly data from March, 1992 to December, 2006 for nominal money supply, real income, short-term nominal interest rate in both China and the US and nominal exchange rate yuan/dollar, and for inflation rate\(^2\) in both countries the sample period is from June, 1992 to March, 2007. The nominal exchange rate yuan/dollar and all American data are from FEDERAL RESERVE BANK OF ST.LOUIS. And all Chinese data are from the CEInet Industry Database. For both countries we use M1 to measure money supply, GDP to measure real income, and CPI to calculate inflation rate. And we use the 1-year constant maturity treasury bill interest rate for the US, and 1-year deposit interest rate for China to measure nominal interest rate. All variables except for inflation rate in both countries are measured in log-levels.

3.2. Unit Root Test

Our first step in testing the long-run monetary model is to examine the integration properties of \(s_t\), \((m_t - m_t^*)\), \((y_t - y_t^*)\), \((\pi_{t+1} - \pi_{t+1}^*)\) and \(R\). We use the standard Augmented Dickey-Fuller (ADF) unit root tests. In Table 1 we report the results of the ADF test for our data.

<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF</th>
<th>ADFd</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_t) (trend and intercept)</td>
<td>-2.391194</td>
<td>-7.494444*</td>
<td>I(1)</td>
</tr>
<tr>
<td>((m_t - m_t^*)) (trend and intercept)</td>
<td>-2.773718</td>
<td>-3.985749*</td>
<td>I(1)</td>
</tr>
<tr>
<td>((y_t - y_t^*)) (trend and intercept)</td>
<td>-2.971857</td>
<td>-4.165268*</td>
<td>I(1)</td>
</tr>
<tr>
<td>((\pi_{t+1} - \pi_{t+1}^*)) (intercept)</td>
<td>-2.103318</td>
<td>-4.094599*</td>
<td>I(1)</td>
</tr>
<tr>
<td>(R) (intercept)</td>
<td>-3.041309</td>
<td>-5.660083*</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

Note: * indicates significance at 1 percent levels. ADF and ADFd indicate ADF statistic for variables and their first differential. (intercept) indicate that test allows for stationarity around a constant, (trend and intercept) indicate that test allows for stationarity around a constant and a linear trend.

Based on the unit root test results in Table 1, we conclude that all variables are \(I(1)\). So the long-run flexible price monetary model (16) requires a cointegrating relationship between \(s_t\), \((m_t - m_t^*)\), \((y_t - y_t^*)\) and \(R\). And the model (12) requires a cointegrating relationship between \(s_t\), \((m_t - m_t^*)\), \((y_t - y_t^*)\) and \((\pi_{t+1} - \pi_{t+1}^*)\).

3.3. Johansen Cointegration Test

Following the unit root tests, we then investigate cointegration using the Johansen maximum likelihood approach (1990). The Johansen method is an essential tool for estimating cointegration in a multivariate system and tests for the number of cointegrating relationships in a VAR

\(^2\) Under rational expectation, we substitute real inflation rate for the expected inflation rate, that is, \(E_t\pi_{t+1} = \pi_{t+1}\) and \(E_t\pi_{t+1}^* = \pi^*_{t+1}\).
system\textsuperscript{[12]}. Because Johansen method is sensitive to small sample bias, we give the rectified small sample critical value which is put forward by Cheung and Lai(1993)\textsuperscript{[13]}. Table 2 reports the cointegration statistics using the Johansen technique.

According to both trace statistics and max-eigenvalue statistics, there are two cointegrating relationships between $s_t$, $(m_t - m_t^*)$, $(y_t - y_t^*)$ and $R$. The cointegrating equation in accordance with the monetary model is,

$$s_t = 2.516858 - 0.11174(m_t - m_t^*) + 0.142657(y_t - y_t^*) - 0.282041R + \mu_t$$

(19)

Log likelihood $= 284.3174$.

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Trace Statistic</th>
<th>Critical value</th>
<th>Max-Eigen Ssa Statistic</th>
<th>Critical value</th>
<th>Ssa</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>110.0157</td>
<td>54.07904</td>
<td>57.94183</td>
<td>62.52245</td>
<td>30.63009</td>
</tr>
<tr>
<td>At most 1</td>
<td>47.49322</td>
<td>35.19275</td>
<td>37.7065</td>
<td>27.66339</td>
<td>23.89245</td>
</tr>
<tr>
<td>At most 2</td>
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<td>9.81916</td>
<td>5.865323</td>
<td>9.81916</td>
</tr>
</tbody>
</table>

Note: *denotes rejection of the hypothesis at the Ssa 0.05 level. Both Trace test and Max-eigenvalue test indicate 2 cointegrating eqns at the Ssa 0.05 level.

Where $\mu_t$ is the residual and standard errors are given in parentheses. $\mu_t$ is stationary, and sum of squared residual equals 5.044559. According to the cointegrating equation, the cointegrating coefficients of relative money supply, relative real income and $R$ are all insignificantly different to zero, and none of them has correct signs as expected by theory. Therefore, although we find evidence of cointegration among nominal exchange rate, relative money supply, relative real income and $R$, the estimated cointegrating vector itself fails to conform to the monetary model. So the long-run flexible price monetary model (16) for yuan/dollar exchange rate does not hold\textsuperscript{[14]}

To make comparison, we also use Johansen method to test the flexible price monetary model (12). Table 3 reports the cointegration statistics using the Johansen technique.

According to both trace statistics and max-eigenvalue statistics, there are two cointegrating relationships between $s_t$, $(m_t - m_t^*)$, $(y_t - y_t^*)$ and $(\pi_{t+1} - \pi_{t+1}^*)$. And the cointegrating equation in accordance with the monetary model is

$$s_t = 2.450146 - 0.029207(m_t - m_t^*) + 0.102991(y_t - y_t^*) - 8.1076(\pi_{t+1} - \pi_{t+1}^*) + \mu_t$$

(20)

Our result is similar to MacDonald and Taylor (1994) and Cushman (2000). Their results showed that there are no support for monetary model in US-UK data and US –Canadian data.
Log likelihood = 503.4196.

<table>
<thead>
<tr>
<th>Hypothesized</th>
<th>Trace 0.05</th>
<th>5%</th>
<th>Max-Eigen 0.05</th>
<th>5%</th>
<th>Ssa Statistic</th>
<th>Critical value</th>
<th>Ssa Critical value</th>
<th>Ssa</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of CE(s)</td>
<td>Statistic</td>
<td>Critical value</td>
<td>Ssa Statistic</td>
<td>Critical value</td>
<td>Ssa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>109.5307*</td>
<td>54.07904</td>
<td>57.94183</td>
<td>57.14226*</td>
<td>28.58808</td>
<td>30.63009</td>
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</tr>
<tr>
<td>At most 1</td>
<td>52.38845*</td>
<td>35.19275</td>
<td>37.70652</td>
<td>31.66606*</td>
<td>22.29062</td>
<td>23.89245</td>
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<tr>
<td>At most 2</td>
<td>20.72239</td>
<td>20.26184</td>
<td>21.70911</td>
<td>15.14025</td>
<td>15.89210</td>
<td>17.02725</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: * denotes rejection of the hypothesis at the Ssa 0.05 level. Both Trace test and Max-eigenvalue test indicate 2 cointegrating eqns at the Ssa 0.05 level.

Where $\mu_t$ is the residual and standard errors are given in parentheses. $\mu_t$ is stationary, and sum of squared residual equals 3.173749. According to the cointegrating equation, all cointegrating coefficients except that of relative inflation rate are insignificantly different from zero, and none of them has correct signs as expected by the theory. Therefore, the long-run flexible price monetary model (12) for yuan/dollar exchange rate does not hold too.

Based on the above testing results of the two flexible price monetary models, we can draw two conclusions. First, although there are evidence of cointegrating relationships between nominal exchange rate and fundamentals, there is little support in the cointegrating coefficient estimates for the flexible price monetary model for yuan/dollar exchange rate. Second, the equation (20) is generally better than the equation (19), for sum of squared residual, the former is smaller than that of the latter, and log likelihood of the former is larger than that of the latter. So there is no transitive impact of different forms of interest rate differential on the flexible price monetary model.

4. Conclusion

In this paper, two flexible price monetary models for yuan/dollar exchange rate, which are separately derived from the generalized monetary model with log-level interest rate differential and that with interest rate differential, was tested to investigate whether the impact of different forms of interest rate differential on the generalized monetary model can pass on to the flexible price monetary model.

We use Johansen maximum likelihood method to test the two long-run flexible price monetary models for yuan/dollar exchange rate. The results show that, there is little support in the cointegrating coefficient estimates for both flexible price monetary models for yuan/dollar exchange rate. However, the model based on the generalized monetary model with interest rate differential is generally better than that based on the generalized monetary model with log-level interest rate differential in the light of sum of squared residual and log likelihood statistics. Therefore, we conclude that there is no transitive impact of different forms of interest rate differential on the flexible price monetary model.
differential on the flexible price monetary model.

Finally, as for the flexible price monetary model unfit for interpreting yuan/dollar exchange rate determination, it is mainly because the practical situation in China could not satisfy the strict assumption of flexible price monetary model, such as China’s pegged exchange rate system, restrict capital control, great difference in economic structure and price system etc with the US. Additionally, the great increase in trade account surplus and capital account surplus recently in China exerts great pressure to the appreciation of the yuan. And obviously this can not be explained by the flexible price monetary model.

References:


Multi-period Bank Hedging with Interest Rate Futures*

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(Received December 16, 2008)

Abstract: In this paper, a model for multi-period bank hedging with interest rate futures is set up. Formulas for the optimal dynamic multi-period bank and static bank hedge ratio are derived. The described model offers the potential benefits of: (1) although these formulas are developed for the case of direct sheet balance multi-period hedging, the framework used is sufficiently flexible so that these formulas can be applied to bank loan or deposit multi-period hedging situations respectively. (2) Periodic modification and updating of the interest rate futures position, as suggested by interest rates, throughout the bank hedging horizons. (3) This paper examines a situation in which the return of loan, the interest rate of deposit and the equity capital of bank, and interest rate futures prices are cointergrated, Multi-period bank hedging formulas are derived under three-dimensional stochastic volatility model. However, empirical research is required for validating this model.

Key words: interest rate futures; multi-period bank hedging; stochastic volatility model

1. Introduction

In the field of risk management of banks many situations call for hedging the gap between rate-sensitive assets and liabilities with the interest rate futures contracts. This study focuses on a multi-period bank hedging optimal strategies to the problem of balance sheet mismatching: interest rate futures are used to hedge the gap. To hedge the risk of an increase interest rates, futures are purchased or sold in an amount that depends on interest rate risk.

Using interest rate futures allows a bank to respond quickly to changes in its economic environment and to continue making its sheet balance. However, There is evidence that the percentage of banks currently using interest rate futures is quite small. Explanations for the

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lack of futures trading by banks include current bank regulations, the use of cash market alternatives to futures trading, or lack of research on the specific practice and usefulness of futures hedging. Because interest rate markets are relatively new, especially to China, increased bank participation in these markets may result from a greater understanding of the optimal bank use and resulting effectiveness of interest rate futures.

Most of the literature which links interest rate uncertainty to hedging decisions by banks is based on the assumption that the bank is concerned with a single period. The conventional portfolio-choice method, which was applied to single period hedging problem faced by a bank, was developed by Ederington (1979)[1]. Koppenhaver (1984, 1985) demonstrate the importance of taking into account both price and quantity risk in a bank’s hedging decision[2,3]. And Morgan, Shone and Smith (1988) applied Koppenhaver model to evaluating the single period hedging performance of various banks[4]. Broll (1999) examines a situation in which hedging may actually increase a bank’s exposure to risk and investigates optimal single period hedging decision in the presence of basis risk in the interest rate futures market[5]. Mun and Morgan (2003) investigates dynamics hedge ratio of a two-period perspective for the US banking system composed of banks with exposure to domestic interest rate and foreign exchange risks[6].

However, these single period alternative bank-hedging models have several limitations. Firstly, these models are less sensitive to the interest rate’s variation. Secondly, these hedging models are myopic. When the joint distribution of spot and futures price changes has a predictable component, myopic hedging models are sub-optimal when the hedger’s horizon extends over multiple periods. Finally, realism suggest, in the futures markets, a futures position is marked to the market on a daily basis (i.e., the losses or gains from a futures position are calculated and accounted for daily), a multi-period bank hedging consideration seems more appropriate.

To correct the shortcomings of the existing literature, this article examines multi-period bank hedging optimal strategies to the problem of balance sheet mismatching: interest rate futures are used to hedge the gap. We derived formulas for the optimal dynamic multi-period bank and static bank hedge ratio. There are three potential advantages of the multi-period bank hedging model. Firstly, although these formulas are developed for the case of direct sheet balance multi-period hedging, the framework used is sufficiently flexible so that these formulas can be applied to bank loan or deposit multi-period hedging situations respectively. Secondly, periodic modification and updating of the interest rate futures position, as suggested by interest rates, throughout the bank hedging horizons. Finally, this paper examines a situation in which the interest rate of loan, deposit and the equity capital of bank and interest rate futures are cointergrated; multi-period bank hedging formulas are derived under stochastic volatility model.

This paper is structured as follows. In Section 2, the theory of a multi-period bank hedging under uncertainty is presented. In Section 3, the multi-period bank-hedging model is set up, and the main results are derived and discussed. In Section 4, the stochastic volatility model
(SV model) was considered to solve the multi-period bank hedging ratios. We get some main results in the presence of a co-integration relationship. We conclude the paper by some final remarks in Section 5.

2. The Theory of Multi-period Bank Hedging

We present the principle of the multi-period bank hedging with interest rate futures. The bank decides on how much to invest the interest rate futures position in different times. To the extent that the issue of multi-period bank hedging arises from the random market interest rate, and the bank should try to reduce the interest rate risk by undertaking hedging transactions at different time points.

The purpose of this paper is to examine a multi-period bank hedging optimal strategies to the problem of balance sheet mismatching: interest rate futures are used to hedge the gap. We presume that the bank's balance sheet is composed of the equity capital of the bank, loans and deposits. As for the equity capital of the bank, it has to satisfy the capital adequacy requirement by regulation. It is assumed to be fixed over the planning horizon. Uncertainty in loan rates arises because the bank commits to a line of credit but knows neither the quantity of the line that will be taken down by the borrower nor the rate of interest it will earn on the loan. Therefore, it is uncertain about the precise rate of return it will earn on the loan. The interest rate of deposits that the bank will be uncontrollable in the long run, which is the same as bank’s loans. Therefore it is uncertain, because the bank is assumed to be a “rate setter” in markets such as the demand and savings deposit markets.

However, it may be necessary to satisfy the balance sheet constraint. That is, it is necessary to keep deposits plus the equity capital of the bank equals to loans. The bank is assumed to operate as a quantity setter in the interest rate futures markets. The interest rate futures market is assumed to be perfect. If the volume of deposits that the bank obtains plus its capital base is insufficient to finance the revealed quantity of loans demanded, the bank must purchase more interest futures contract in the interest rate futures market at a competitive but uncertain cost.

The Theory of multi-period bank hedging is illustrated in Figure 1. Recognizing the existence of uncertainty, the bank may, at the initial time point 0, enter $f_0$ futures contract if it desires to hedge the rate uncertainties connected with its assets and liabilities. However, the loan’s return, deposit and the equity capital’s interest rates may change very quickly at time point 1. The bank should modify his futures position to $f_1$, thus the futures position is updated. In this way, the bank can adjust the futures position through to time $T$. All uncertainty is resolved at time point $T$, the futures position are Liquidated in the interest rate futures market. However, it is necessary to keep the structure of the balance sheet at time point $T$. 

3. A Model of Bank Hedging

According to the theory of multi-period bank hedging, a model of bank hedging is set up. The model used here is heavily related to the model developed by Broll (1999)\textsuperscript{[5]}. For simplicity, initial margins and variation margins are ignored.

The balance sheet constraint at time $T$ is $L = D + B$, where

$L$ is the actual quantity of loans demanded at time $T$ under the line-of-credit agreement,

$D$ is the quantity of deposits received at time $T$ by the bank,

$B$ is the equity capital of the bank, which is assumed to be fixed over the planning horizon, although it has to satisfy the following capital adequacy requirement by regulation: $B \geq \tau D$, where $\tau$ is the required minimum capital deposit ratio.

Profits are obtained from the loan revenue less the cost of debt in the two markets in which the bank borrows plus an gain or loss in the futures market resulting from the bank’s futures position. Assuming that the capital requirement constraint, $B \geq \tau D$, is binding, the profit function can therefore be written as:

$$
\Pi_T = LR_T^L - DR_T^D - BR_T^B - \sum_{t=0}^{T-1} f_t(r_{t+1} - r_t)
$$

where

$R_T^L$ represents the return of the loans,

$R_T^D$ is the interest rate on the deposits,

$R_T^B$ is the interest rate on the equity capital of the bank,

$r_t$ is the interest rate futures at which the contract is initially undertaken at time $t$,

$f_t$ is the quantity of interest rate futures contracts that the bank enters at time $t$ ($f_t > 0$ corresponds to a short position and $f_t < 0$ corresponds to a long position).

Upper case letters denote variables that are random when viewed from time point. The balance sheet constraint can be used to express $B$ in terms of the other quantities. Substituting
back into the profit function gives:

$$\Pi_T = L(R^L_T - R^B_T) - D(R^D_T - R^B_T) - \sum_{t=0}^{T-1} f_t(r_{t+1} - r_t)$$  \hspace{1cm} (2)$$

where $R^L_T = R^L_T - R^B_T$, the spread between the loan rate and the interest rate on the equity capital of the bank. And $R^D_T = R^D_T - R^B_T$, the spread between the deposits rate and the interest rate on the equity capital.

The equation (2) can be written as:

$$\Pi_T = LR^L_T - DR^D_T - \sum_{t=0}^{T-1} f_t(r_{t+1} - r_t)$$  \hspace{1cm} (3)$$

A decision is made to hedge this position for the next $T$ periods using interest rate futures contract. We pursue the position $f_t$ taken in the futures market at time $t$ under the adopted hedging strategy.

We assume that the objective of the bank is to minimize the variance of the profits at time $T$. Suppose the bank decides to pursue a dynamic multi-period bank hedging strategy. The optimal multi-period hedge is determined using backward iteration method. At time $T - 1$, the conditional variance of the preceding profit is

$$\text{Var}_{(T-1)}(\Pi_T) = L^2\text{Var}_{(T-1)}(R^L_T) + D^2\text{Var}_{(T-1)}(R^D_T) + f^2_{T-1}\text{Var}_{(T-1)}(r_T)$$

$$-2f_{T-1}(LCov(R^L_T, r_T) - DCov(R^D_T, r_T)) - 2LD\text{Cov}(R^L_T, R^D_T)$$  \hspace{1cm} (4)$$

which is minimized by choosing a hedge ratio of

$$f^*_{T-1} = \frac{(LCov(R^L_T, r_T) - DCov(R^D_T, r_T))}{\text{Var}_{(T-1)}(r_T)}$$  \hspace{1cm} (5)$$

Next, consider the bank’s problem at time $T - k$. Assume that the bank has already solved for the optimal hedge ratios $f^*_{T-2}$, $f^*_{T-3}$, $f^*_{T-4}$, $\ldots$, $f^*_{T-k+1}$. These hedge ratios may depend on price information after $T - k$. In such situations, these hedge ratios must be treated as stochastic variables at time $T - k$. The conditional variance of the bank’s profit is

$$\text{Var}_{(T-k)}(\Pi_T) = \text{Var}_{(T-k)}(LR^L_T - DR^D_T - \sum_{t=T-k+1}^{T-1} f^*_t(r_{t+1} - r_t)) + f^2_{T-k}\text{Var}_{(T-k)}(r_{T-k+1})$$

$$- 2f^*_{T-k}\text{Cov}(r_{T-k+1}, LR^L_T - DR^D_T - \sum_{t=T-k+1}^{T-1} f^*_t(r_{t+1} - r_t))$$  \hspace{1cm} (6)$$

Because the hedge ratios $f^*_{T-2}$, $f^*_{T-3}$, $f^*_{T-4}$, $\ldots$, $f^*_{T-k+1}$ occur within the conditional covariance operator, they can be either stochastic or deterministic at time $T - k$. The conditional
variance is minimized by choosing a hedge ratio of

\[ f_{T-k}^* = \frac{\text{Cov}(r_{T-k+1}, LR_{T-k}^{LB} - DR_{T-k}^{BB}) - \sum_{t=T-k+1}^{T-1} f_t^*(r_{t+1} - r_t)}{\text{Var}(r_{T-k})} \]

\[ = \frac{\text{LCov}(r_{T-k+1}, R_{T-k}^{LB}) - DR_{T-k}^{BB}}{\text{Var}(r_{T-k})} \]

\[ - \sum_{t=T-k+1}^{T-1} \frac{f_t^*[\text{Cov}(r_{T-k+1}, r_{t+1}) - \text{Cov}(r_{T-k+1}, r_t)]}{\text{Var}(r_{T-k})} \]  \quad (7)

By using the equation (5) to compute the hedge ratio in the final period and then applying the equation (7) recursively to solve the hedge ratio in each period, the optimal dynamic hedge can be completely determined.

Thus, making use of the equations (5) and (7), we get:

**Proposition 1** For a T-period bank hedging horizon, the optimal hedge ratio at time \( k(0 \leq k \leq T-1) \), \( f_{T-k}^* \), satisfies the following recursive relation:

\[ f_{T-k}^* = \frac{\text{LCov}(r_{T-k+1}, R_{T-k}^{LB}) - DR_{T-k}^{BB}}{\text{Var}(r_{T-k})} \]

\[ - \sum_{t=T-k+1}^{T-1} \frac{f_t^*[\text{Cov}(r_{T-k+1}, r_{t+1}) - \text{Cov}(r_{T-k+1}, r_t)]}{\text{Var}(r_{T-k})} \]  \quad (8)

Proposition 1 gives multi-period bank hedging optimal strategies to the problem of balance sheet mismatching: interest rate futures are used to hedge the gap.

If assume \( D = B = 0 \) or \( L = B = 0 \), following proposition 1 derivative process, we can get Proposition 2 and Proposition 3.

**Proposition 2** If the bank’s loans \( L = B = 0 \), that is, the bank only hedging deposit, the optimal T-period hedge ratios at time \( k(0 \leq k \leq T-1) \), \( f_{T-k}^* \), is given as follows:

\[ f_{T-k}^* = \frac{-DR_{T-k}^{BB}}{\text{Var}(r_{T-k})} \]

\[ - \sum_{t=T-k+1}^{T-1} \frac{f_t^*[\text{Cov}(r_{T-k+1}, r_{t+1}) - \text{Cov}(r_{T-k+1}, r_t)]}{\text{Var}(r_{T-k})} \]  \quad (9)

**Proposition 3** If the bank’s deposits \( D = B = 0 \), that is, the bank only hedging loans, the optimal T-period hedge ratios at time \( k(0 \leq k \leq T-1) \), \( f_{T-k}^* \), is given as follows:

\[ f_{T-k}^* = \frac{\text{LCov}(r_{T-k+1}, R_{T-k}^{LB})}{\text{Var}(r_{T-k})} \]

\[ - \sum_{t=T-k+1}^{T-1} \frac{f_t^*[\text{Cov}(r_{T-k+1}, r_{t+1}) - \text{Cov}(r_{T-k+1}, r_t)]}{\text{Var}(r_{T-k})} \]  \quad (10)

Proposition 2 and Proposition 3 show that the bank could be hedging loan or deposit with interest rate futures using the equation (9) or (10) respectively.

However, in many situations, the bank may prefer to use a static hedging strategy. Static hedging involves much lower transaction costs than multi-period bank hedging. Additionally,
static hedging strategies are less sensitive to estimation and modeling errors than dynamic multi-period hedging strategies. As a result, static hedges may frequently outperform dynamic multi-period hedges in practice.

Suppose the bank decides to pursue a static hedging strategy. When the hedge ratio is held constant, the profit of the bank simplifies to

$$\Pi_T = LR_T^L - DR_T^D - BR_T^B - f(r_T - r_0) = LR_T^{LB} - DR_T^{DB} - f(r_T - r_0) \quad (11)$$

The conditional variance of this profit is

$$\text{Var}_0(\Pi_T) = L^2\text{Var}_0(R_T^{LB}) + D^2\text{Var}_0(R_T^{DB}) + f^2\text{Var}_0(r_T)$$

$$- 2f(L\text{Cov}(R_T^{LB}, r_T) - D\text{Cov}(R_T^{DB}, r_T)) - 2LD\text{Cov}(R_T^{LB}, R_T^{DB}) \quad (12)$$

which is minimized by setting the hedge ratio to

$$f^* = \frac{(L\text{Cov}(R_T^{LB}, r_T) - D\text{Cov}(R_T^{DB}, r_T))}{\text{Var}_0(r_T)} \quad (13)$$

We conclude as follows.

**Proposition 4** For the bank static hedging, the optimal hedge ratio, $f^*$, satisfies the following equation:

$$f^* = \frac{(L\text{Cov}(R_T^{LB}, r_T) - D\text{Cov}(R_T^{DB}, r_T))}{\text{Var}_0(r_T)} \quad (14)$$

This static hedge is identical to the conventional hedge when either $R_T^{LB}$ and $r_T$ changes or $R_T^{DB}$ and $r_T$ changes are independently and identically distributed through time.

However, The equation (14) single period bank-hedging model have several limitations. Firstly, these models are less sensitive to the interest rate’s variation. Secondly, these hedging models are myopic. When the joint distribution of the spots and futures price changes has a predictable component, myopic hedging models are sub-optimal when the hedger’s horizon extends over multiple periods. Finally, realism suggest, in the futures markets, a futures position is marked to the market on a daily basis (i.e., the losses or gains from a futures position are calculated and accounted for daily), a multi-period bank hedging consideration seems more appropriate.

4. The Multi-period Bank Hedging Under Stochastic Volatility Model

The $T$-period bank hedging the equations (8)–(10) are nontrivial. As a result, simulation techniques are required in calculating multi-period hedge ratios. In this section, we estimate the optimal hedge ratios using a multivariate stochastic volatility (SV) model for multi-period bank hedging.

4.1. The SV Model
Optimal multi-period bank hedge ratios can be statistically estimated by running the multivariate SV model for performance evaluation of hedging. In modeling the stochastic process for the return of loans, the interest rate of deposits and the equity capital of bank and futures prices, we incorporate SV model (see Lien and Wilson, 2001; Lien, 2004)[7,8]. The reason is that most researchers agree that both spot and futures prices tend to contain a unit root (see Lien and Luo, 1993, 1994; Lien and Wilson, 2001; Lien, 2004). It also means that the presence of a cointegration relationship is not unanimously supported. Herein, we allow for the possibility of a cointegration relationship and consider the following three-dimensional SV model that contains only variables of the first lag:

\[ R_t^{LB} = a_0a_1\theta_{t-1} + a_2R_{t-1}^{LB} + a_3R_{t-1}^{DB} + a_4r_{t-1} + \varepsilon_{1,t} \] (15.a)

\[ R_t^{DB} = b_0b_1\theta_{t-1} + b_2R_{t-1}^{LB} + b_3R_{t-1}^{DB} + b_4r_{t-1} + \varepsilon_{2,t} \] (15.b)

\[ r_t = c_0c_1\theta_{t-1} + c_2R_{t-1}^{LB} + c_3R_{t-1}^{DB} + c_4r_{t-1} + \varepsilon_{3,t} \] (15.c)

\[ \varepsilon_{i,t} = \exp(h_{i,t})u_{i,t}, \quad i = 1, 2, 3 \] (16.a)

\[ h_{i,t} = \alpha_i + \beta_i h_{i,t-1} + \eta_{i,t}, \quad i = 1, 2, 3 \] (16.b)

where \( u_t = (u_{1,t}, u_{2,t}, u_{3,t})^T \) forms a sequence of independent three-dimensional normal random vectors such that \( E(u_t) = (0, 0, 0)^T \) and

\[ \text{Var}(u_t) = \begin{pmatrix}
1 & \text{Cov}(u_{1,t}, u_{2,t}) & \text{Cov}(u_{1,t}, u_{3,t}) \\
\text{Cov}(u_{2,t}, u_{1,t}) & 1 & \text{Cov}(u_{2,t}, u_{3,t}) \\
\text{Cov}(u_{3,t}, u_{1,t}) & \text{Cov}(u_{3,t}, u_{2,t}) & 1
\end{pmatrix} \]

\[ \Delta \begin{pmatrix}
1 & \rho_{12} & \rho_{13} \\
\rho_{21} & 1 & \rho_{23} \\
\rho_{31} & \rho_{32} & 1
\end{pmatrix}, \quad \rho_{ij} = \rho_{ji}; i \neq j; i, j = 1, 2, 3 \]

Let \( \eta_t = (\eta_{1,t}, \eta_{2,t}, \eta_{3,t})^T \). Then \( \eta_t \) is a three-dimensional normal vector with \( E(\eta_t) = (0, 0, 0)^T \) and \( \text{Var}(\eta_t) = \Sigma \). For any \((t, s)\), \( \text{Cov}(\eta_{i,t}, u_{j,s}) = 0, i, j = 1, 2, 3 \). Finally, \( \theta_t = r_t - l_1 R_t^{LB} - l_2 R_t^{DB} \) is the proposed cointegration relationship, where the \( l_1 \) and \( l_2 \) are constant coefficients.

Following Engle and Granger (1987)[11], the cointegration variable \( \theta_{t-1} \) is incorporated in the mean equations with \( a_1 \neq 0, b_1 \neq 0 \) and \( c_1 \neq 0 \). While the equations (15)–(17) contain only variables of the first lag, our analysis can be extended to the case with higher-order lags. The equations (18) and (19) describe SV in \( R_t^{LB} \), \( R_t^{DB} \) and \( r_T \). A general formulation is as follows:

\[ h_t = \alpha + \beta h_{t-1} + \eta_t \] (17)

where \( h_t = (h_{1,t}, h_{2,t}, h_{3,t})^T \), \( \alpha = (a_0, b_0, c_0)^T \), \( \eta_t = (\eta_{1,t}, \eta_{2,t}, \eta_{3,t})^T \) and \( \beta \) is a \((3 \times 3)\) matrix. By assuming that \( \beta \) is a diagonal matrix, the equation (16) is reduced to the equations (15.a)–(15.c).
4.2. The Optimal Multi-period Bank Hedging

To examine this conjecture and its implications for multi-period bank hedging, we have characterized the multi-period minimum-risk hedge strategy within the SV framework. We now characterize $f^*_T$ under the SV assumptions. From the equations (15.a)–(17), we can derive

$$
\text{Cov}(R_{TB}^L, r_T) = \text{Cov}[\exp(h_{1,T})u_{1,T}, \exp(h_{3,T})u_{3,T}]
$$

$$
= \text{Cov}([1, 0, 0][\alpha + \beta h_{T-1} + \eta_T]u_{1,T}, \exp([0, 0, 1][\alpha + \beta h_{T-1} + \eta_T])u_{3,T}]
$$

$$
= E[u_{1,T}u_{3,T}]E[\exp([1, 0, 1][\alpha + \beta h_{T-1} + \eta_T])]
$$

$$
= \rho_{13} \exp([1, 0, 1][\alpha + \beta h_{T-1}]) + [[1, 0, 1]\Sigma[1, 0, 1]^T]/2
$$

Similarly, we have

$$
\text{Cov}(R_{TB}^D, r_T) = \rho_{23} \exp([0, 1, 1][\alpha + \beta h_{T-1}]) + [[0, 1, 1]\Sigma[0, 1, 1]^T]/2
$$

In the above derivation, we use the properties that $u_T$ and $\eta_T$ are stochastically independent and that $u_T$ and $\eta_T$ is a three-dimensional normal vector. Similarly, we can find that:

$$
\text{Var}(r_T) = E(\exp(2h_{3,T})u_{3,T})
$$

$$
= \exp([0, 0, 2][\alpha + \beta h_{T-1}] + [0, 0, 2]\Sigma[0, 0, 2]^T)/2
$$

Substituting the equations (14)–(16) into the equation (5), we can get:

$$
f^*_T = \frac{(L\text{Cov}(R_{TB}^L, r_T) - D\text{Cov}(R_{TB}^D, r_T))}{\text{Var}(r_T)}
$$

$$
= \frac{(L\rho_{13} \exp([1, 0, 1][\alpha + \beta h_{T-1}]) + [[1, 0, 1]\Sigma[1, 0, 1]^T]/2) - D\rho_{23} \exp([0, 1, 1][\alpha + \beta h_{T-1}]) + [[0, 1, 1]\Sigma[0, 1, 1]^T]/2))}{\exp([0, 0, 2][\alpha + \beta h_{T-1}] + [0, 0, 2]\Sigma[0, 0, 2]^T)/2}
$$

Note that $f^*_T$ depends only upon $h_{T-1}$, which, in turn, is a function of $\{h_1, h_2, \ldots, h_{T-1}\}$. Because $h_t$ and $u_t$ are stochastically independent, $f^*_T$ is independent of $r_{k-1}$ for any $k$. By induction, we establish the following proposition.

**Proposition 5** Under the SV assumptions, the optimal multi-period hedge ratio $f^*_{T-k}$ is stochastically independent of $r_k$ for any $k$.

The stochastic independence results simplify the calculation of multi-period bank hedge ratios. More specifically, $f^*_{T-2}$ can be written as follows:

$$
f^*_T = \frac{L\text{Cov}(r_{T-1}, R_{TB}^L) - D\text{Cov}(r_{T-1}, R_{TB}^D)}{\text{Var}(r_{T-1})} = \frac{f^*_{T-1}\text{Cov}(r_{T-1}, (r_T - r_{T-1}))}{\text{Var}(r_{T-1})}
$$

$$
= \frac{L\text{Cov}(r_{T-1}, R_{TB}^L) - D\text{Cov}(r_{T-1}, R_{TB}^D)}{\text{Var}(r_{T-1})} = \frac{f^*_{T-1}\text{Cov}(r_{T-1}, r_T)}{\text{Var}(r_{T-1})} - 1
$$

From the equations (16) and (17), we derive

$$
\text{Var}(r_{T-2}(r_T - r_{T-1}) = \exp([0, 0, 2][\alpha + \beta h_{T-2}] + [0, 0, 2]\Sigma[0, 0, 2]^T)/2
$$

$$
= \text{Var}(r_{T-2})
$$
To calculate Cov\(\{r_{T-1}, R^B_T\}\), Cov\(\{r_{T-1}, R^{DB}_T\}\) and Cov\(\{r_{T-1}, r_T\}\), note that from the equations (15.a)–(15.c), with the cointegration relation assumption, we have

\[
\theta_t = r_t - l_1 R^B_t - l_2 R^{DB}_t \\
= (c_0 - a_0 l_1 - b_0 l_2) + (c_1 - a_1 l_1 - b_1 l_2)\theta_{t-1} + (c_2 - a_2 l_1 - b_2 l_2)R^B_{t-1} \\
+ (c_3 - a_3 l_1 - b_3 l_2)R^{DB}_{t-1} + (c_4 - a_4 l_1 - b_4 l_2)r_{t-1} + (\varepsilon_{3,t} - l_1 \varepsilon_{1,t} - l_2 \varepsilon_{2,t})
\]  

(24)

Define \(x_t = (R^B_t, R^{DB}_t, r_t, \theta_t)^T\) and \(\omega_t = [\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}, (\varepsilon_{3,t} - l_1 \varepsilon_{1,t} - l_2 \varepsilon_{2,t})]^T\). Then the equations (15.a)–(15.c), and (20) form a vector equation system

\[
x_t = C_0 + C_1 x_{t-1} + \omega_t
\]

(25)

where \(C_0 = [a_0, b_0, c_0, (c_0 - l_1 a_0 - l_2 b_0)]^T\) and

\[
C_1 = \begin{pmatrix}
a_1 & a_2 & a_3 & a_4 \\
b_1 & b_2 & b_3 & b_4 \\
c_1 & c_2 & c_3 & c_4 \\
c_1 - l_1 a_1 - l_2 b_1 & c_2 - l_1 a_2 - l_2 b_2 & c_3 - l_1 a_3 - l_2 b_3 & c_4 - l_1 a_4 - l_2 b_4
\end{pmatrix}
\]

(26)

From the equation (13), we have

\[
Cov(x_{T-1}, x_T) = Var_{T-2}(\omega_{T-1})C_1^T
\]

(27)

\[
Cov(r_{T-1}, R^B_T) = [0, 0, 1, 0]\text{Cov}(x_{T-1}, x_T)[1, 0, 0, 0]^T
\]

(28)

\[
Cov(r_{T-1}, R^{DB}_T) = [0, 0, 1, 0]\text{Cov}(x_{T-1}, x_T)[0, 1, 0, 0]^T
\]

(29)

\[
Cov(r_{T-1}, r_T) = [0, 0, 1, 0]\text{Cov}(x_{T-1}, x_T)[0, 0, 1, 0]^T
\]

(30)

using the property that \(\{\omega_t\}\) is a sequence of stochastically independent random vectors. Upon combining the equation (19) with the equations (23)–(26), \(f^*_T\) can be calculated.

\[
f^*_{T-2} = \frac{L\text{Cov}(r_{T-1}, R^B_T) - D\text{Cov}(r_{T-1}, R^{DB}_T)}{\text{Var}(T-2)(r_{T-1})} - f^*_{T-1}\frac{\text{Cov}(r_{T-1}, r_T)}{\text{Var}(T-2)(r_{T-1})} - 1
\]

\[
= \frac{\begin{vmatrix} L[0, 0, 1, 0]\text{Cov}(x_{T-1}, x_T)[1, 0, 0, 0]^T - D[0, 0, 1, 0]\text{Cov}(x_{T-1}, x_T)[0, 1, 0, 0]^T \\
\text{exp}[\{0, 0, 2\} + \alpha + \beta h_{T-2}] + [0, 0, 2, 0]\Sigma[0, 0, 2]^T / 2
\end{vmatrix}}{\text{Var}(T-2)(r_{T-1})}
\]

\(
- f^*_{T-1}\frac{\begin{vmatrix} [0, 0, 1, 0]\text{Cov}(x_{T-1}, x_T)[0, 0, 1, 0]^T \\
\text{exp}[\{0, 0, 2\} + \alpha + \beta h_{T-2}] + [0, 0, 2, 0]\Sigma[0, 0, 2]^T / 2
\end{vmatrix}}{\text{Var}(T-2)(r_{T-1})} - 1
\)

(31)

In fact, \((f^*_{T-2}, f^*_{T-1})\) is the solution for a two-period minimum-risk hedging problem from time \(T-2\) to \(T\).

The above method can be extended to derive \(f^*_T, k = 3, 4, \ldots, T\)

\[
\text{Var}(T-k)(r_{T-k+1}) = \text{exp}[\{0, 0, 2\} + \alpha + \beta h_{T-k}] + [0, 0, 2, 0]\Sigma[0, 0, 2]^T / 2, \quad k = 3, 4, \ldots, T
\]

(32)

\[
\text{Cov}(x_{T-k+1}, x_t) = \text{Var}(T-k)(\omega_{T-k+1})(C_1^{-(T-k+1)})^T, \quad t = T-k+2, T-k+3, \ldots, T
\]

(33)
Substituting the equations (32)–(36) into the equation (8), we can conclude as follows.

**Proposition 6**  
Under the SV assumptions, the optimal multi-period hedge ratio \( f^*_{T-k} \) is written as follows:

\[
f_{T-k}^* = \frac{LCov(r_{T-k+1}, R^{LB}_T) - DCov(r_{T-k+1}, R^{DB}_T)}{\text{Var}(T-k)(r_{T-k+1})} - \sum_{t=T-k+1}^{T-1} \frac{f^*_t [\text{Cov}(r_{T-k+1}, r_{t+1}) - \text{Cov}(r_{T-k+1}, r_t)]}{\text{Var}(T-k)(r_{T-k+1})} \]  

(37)

In the presence of interest rate risk, which is a function of the variance of the futures price and the correlation between the spots and futures prices, the futures market provides an imperfect hedging mechanism.

5. Conclusions

In this article, the multi-period bank hedging with interest rate futures problem has been examined to solve balance sheet mismatching: interest rate futures are used to hedge the gap. Formulas for the optimal dynamic multi-period bank and static bank hedge ratio are derived. Although these formulas are developed for the case of direct sheet balance multi-period hedging, the framework used is sufficiently flexible so that these formulas can be applied to bank loan or deposit multi-period hedging situations respectively. The bank can adopt the periodic modification and updating of the interest rate futures position, as suggested by interest rates, throughout the bank hedging horizons. Finally, the hedging problem has been examined when the loan’s return, interest rates of deposit and the equity capital of bank and interest rate futures prices follow a variant of three-dimensional stochastic volatility model. However, for the lack of empirical research, our results do not rule out the possible usefulness of SV models for futures market study. Further research in the multi-period bank hedging is required to resolve this issue.

References:


Study on China’s Economic Vulnerability to Energy Import Using Decomposition Method*

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Abstract: Vulnerability means the degree to which that a system is susceptible to suffer damage. This paper focuses on the economic vulnerability to risk of energy import by employing ratio of net energy import to GDP as indicator, and decomposes the vulnerability change into effects of energy import, structure and intensity in order to find out key factors that influence economic security to energy import. Decomposition analysis on China indicates that effect of rising energy import takes more than 90 percent of total vulnerability change during the last 10 years, along with insignificant effect of structural change and intensity decline. International analysis on cross-section data of net energy importers also presents the positive relationship between external energy dependence and economic vulnerability. However, results of America show that long-term effect of energy intensity is much larger than China from 1954 to 2007, which is 70.8% of its total vulnerability change. Experience from developed countries confirms the necessary and validity of improving energy efficiency on depressing economic vulnerability to energy import, which provides lessons for the energy development of China.

Key words: economic vulnerability; energy import; decomposition analysis

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1. Background

As the second largest energy consumption and production country in the world, China’s energy demand has exceeded domestic production since 1992 (see Fig.1.). China became a net oil importer in 1996 and a net energy importer in 1997, while the ratio of oil dependence on import is as high as 45% in 2007, and ratio of total energy dependence on import is more than 10%.

![Energy supply/demand and import/export of China](image)

Fig.1. Energy supply/demand and import/export of China

Increasing external dependence causes growing risk of energy supply, as well as economic vulnerability to energy import. Petroleum related industries take almost 20 percent of China’s industrial value-added, which provide more than 90% fuel of transport and half of agriculture. Hence both disruption of oil supply and sharply rise of oil price will cause serious shock to national economy.

Since the most significant risk of energy supply comes from import in the country’s point of view, it’s necessary to make quantitative analysis on the threat of energy import risk to national economy, which is described as an index of energy vulnerability here. The following paper will focus on the design and analysis of the indicator that represents economic vulnerability to energy import, and find out the key factors that influence energy security of the country.

2. Literatures on Energy Vulnerability

Vulnerability is defined as the degree of loss to a given element at risk (or set of elements) resulting from a given hazard at a given severity level, and the vulnerability of a system is the degree to which that system is unable to cope with selected adverse events. Vulnerability is a context dependent concept since it’s not clear about the source of risk, the subject and type of loss; hence vulnerability research covers a complex, multidisciplinary field including climate studies, disaster and risk management, engineering, geography, information network, security studies, sustainable development, and so on.

The concept of energy vulnerability originated from energy crisis decades ago. Sterman
employs system dynamic model to study the impact of energy transition on economy as early as 1980s\cite{5}, while Clark and Page study on the problem of energy supply on national security\cite{6}. According to the above implications and main subjects under research, there are three most commonly used meanings of vulnerability in field of energy policy and energy economics: the first is economic vulnerability to energy supply, including impact of supply shortage and price fluctuation; the second is environmental vulnerability to energy use, especially climate change promoted by fossil energy consumption which is also one of the most popular issues in international politics\cite{7,8}; the last one is vulnerability of energy supply to risks in production, transportation and import etc.

In view of the common contents of study, researches on vulnerability are usually under the framework of energy security\cite{9}. McCarthy et al. take infrastructure vulnerability as one of the three main concepts of energy security\cite{10}, and construct a comprehensive hierarchy to assess reliability of energy systems, where vulnerability is the degree to which the system is susceptible to disruption. Costantini et al. also take vulnerability as one of the categories to measure energy security along with dependence\cite{11}, which is described by concentration and diversity in physical dimension as well as indicator of energy consumption per GDP in economic term. Similar study has been taken by Kendell on oil dependence and vulnerability as early as 1998\cite{12}, where measures of vulnerability in economic dimension include consumption per GDP and expenditures per GDP.

Since petroleum is the major energy for consumption and international trade in the world, and import is the main source of supply risk which is much more difficult to forecast and eliminate, most studies of energy vulnerability focus on the economic impact of oil import risk and take the ratio of net oil import to GDP as indicator. On research of Ukraine’s oil/natural gas vulnerability\cite{13}, this indicator was broken down into three indicators including ratio of import dependence, ratio of oil/gas to total energy consumption and energy intensity, as explained by Chevalier\cite{14}. Nakawiro and Bhattacharyya add gas price as a multiplicator in the indicator when evaluating the vulnerability of electricity supply due to high gas dependence in Thailand\cite{15}. Gupta employs a complicated index system with three supply risk indicators and four market risk indicators for measuring oil vulnerability\cite{16}, which also takes the ratio of value of oil import to GDP as an important factor, so does WEC on the study of European’s vulnerability to energy crisis\cite{17}.

3. Design and Decomposition of Vulnerability Indicator

3.1. Indicators of Vulnerability

Based on the research described above, net energy import per GDP is the most appropriate indicator for assessing economic vulnerability to energy import, since both energy import and economic growth are embodied in the expression, which could be further calculated as the
product of energy intensity and rate of energy import dependence:

\[ \text{Vulnerability Indicator} = \frac{\text{Net Energy Import}}{\text{GDP}} = \frac{\text{Net Energy Import}}{\text{Energy Consumption}} \times \frac{\text{Energy Consumption}}{\text{GDP}} \]  \hspace{1cm} (1)

According to the classification of different energy types (coal, oil, natural gas and electricity), this indicator can be computed as summation of vulnerability each kinds energy, which could be further decomposed into three factors of import dependence, energy structure and energy intensity. These are also key factors that influence energy security:

\[ \text{VI} = \sum_i \text{VI}_i = \sum_i \frac{\text{NEI}_i}{\text{EC}_i} \]  \hspace{1cm} (2)

where \( \text{VI} \) represents indicator of economic vulnerability to energy import, subscript \( i \) represents the type of energy, \( \text{NEI}_i \) and \( \text{RID}_i \) is net import and rate of dependence on import of energy type \( i \), \( \text{ES}_i \) means the proportion of energy type \( i \) in total primary energy consumption, and \( \text{EI} \) is energy intensity, i.e. energy consumption per GDP.

3.2. Decomposition Approach

Based on the structure of indicator presentation by former equations, change of vulnerability indicator can be decomposed and distributed among a number of influence factors as follows:

\[ \Delta \text{VI} \equiv \text{VI}^T - \text{VI}^0 = \Delta \text{VI}_{dep} + \Delta \text{VI}_{int} = \sum_i \Delta \text{VI}_i = \sum_i (\Delta \text{VI}_{dep,i} + \Delta \text{VI}_{str,i} + \Delta \text{VI}_{int,i}) \]  \hspace{1cm} (3)

where superscripts \( T \) and \( 0 \) represent time period, \( \Delta \text{VI}_{dep} \) is the influence of energy import, \( \Delta \text{VI}_{str} \) is the influence of energy structure and \( \Delta \text{VI}_{int} \) is the influence of energy intensity, subscript \( i \) represents the type of energy.

There are many methods for calculating the decomposition of variables, while the most popular approach nowadays is logarithm mean Divisia index method generalized by Ang which is perfect without residuals\(^{[18]}\), the effects of import and intensity are calculated as follows:

\[ \Delta \text{VI}_{dep} = \text{L}(\text{VI}^T - \text{VI}^0) \ln \left( \frac{\text{RID}^T}{\text{RID}^0} \right) , \quad \Delta \text{VI}_{int} = \text{L}(\text{VI}^T - \text{VI}^0) \ln \left( \frac{\text{ES}^T}{\text{ES}^0} \right) \]  \hspace{1cm} (4)

where function \( \text{L}(x, y) \) is logarithmic average of two positive numbers \( x \) and \( y \) given by

\[ \text{L}(x, y) = \begin{cases} \frac{(x - y)}{\ln x - \ln y} & x \neq y \\ x & x = y \end{cases} \]  \hspace{1cm} (5)
Effects of each factor by energy type $i$ are calculated as follows:

$$
\Delta V_{I_{dep},i} = L(V_{I_i}^T - V_{I_i}^0) \ln \left( \frac{R_{ID_i}^T}{R_{ID_i}^0} \right)
$$

$$
\Delta V_{I_{str},i} = L(EV_{I_i}^T - EV_{I_i}^0) \ln \left( \frac{E_{SI_i}^T}{E_{SI_i}^0} \right)
$$

$$
\Delta V_{I_{int},i} = L(EV_{I_i}^T - EV_{I_i}^0) \ln \left( \frac{E_{IT_i}^T}{E_{IT_i}^0} \right)
$$

(6)

Since this method may produce significant errors if applied on data set containing a large number of zero or small values, as well as negative values\cite{19}, Ang and Liu propose the modified method of dealing with zero-value and negative-value problems subsequently that could be generally applied to any decomposition situation\cite{20,21}. There are indeed some negative values in our study such as net import of coal or natural gas. Since the case is not complicated, they are processed by simple operation of decomposition method.

4. Empirical Analysis

4.1. Decomposition Analysis on China

During the last ten years China’s net energy import has increased from 21 million tons of coal equivalents (mtce) in 1997 to 246 mtce in 2007, the average speed of increase is 28% per year, much more faster than the increase of energy consumption and GDP, while indicator of economic vulnerability to energy import also rises from 2.62 tce / million yuan (RMB) to 12.68 tce / million yuan (2000 constant price). Fig.2. shows the change of China’s vulnerability indicator from 1997 to 2007, as well as results of decomposition analysis which represents the effect by change of energy intensity, energy structure and rate of energy dependence on import. Data used in the analysis are gathered from Statistical Yearbook of NBSC\cite{1,22-28}.

From the result of decomposition, we can find that changes of vulnerability are mostly dominated by increase of energy dependence on import. The accumulative change of vulnerability during the last ten years is 10.06 tce / million yuan, where effect of energy import is 11.97 tce / million yuan, but effect of energy structure is only −1.18 tce / million yuan, and effect of energy intensity is −0.73 tce / million yuan. As matter of fact, increasing import of oil and decreasing export of coal are the main cause of China’s increasing economic vulnerability to energy import, effects of which are 6.61 tce /million yuan and 4.17 tce /million yuan respectively.

Because the proportion of oil in China’s total energy consumption has dropped since 2002, influence of energy structure is negative but much smaller than that of energy import. In the year 2007, ratio of oil, coal, gas and electricity to China’s total primary consumption are 19.7%, 69.5%, 3.5% and 7.3%\cite{27}, extremely imbalanced compared to the average level of the world, structure of which are 35.6%, 28.6%, 23.8% and 12.0%. China is still a net exporter of coal, gas and electricity, accumulated effects of them are all positive to the increase of vulnerability,
especially the fast growing import of oil and sudden decline of coal export. Statistical data shows that coal consumption in the year 2007 is 2.58 billion tons\textsuperscript{[28]}, larger than the production of 2.54 billion tons, which ends the age of China’s massive export of coal. Considering the global trend of energy diversification for energy security as well as increasing import of petroleum and natural gas in China\textsuperscript{[29]}, effect of energy structural change on vulnerability will be positive in the long term. Therefore reducing energy intensity is the only way to restrain China’s economic vulnerability to energy import.

![Graph showing decomposition of China’s vulnerability indicator 1997–2007 (tce/million yuan)](image)

Fig.2. Decomposition of China’s vulnerability indicator 1997–2007 (tce/million yuan)

Although China’s energy intensity decreases 21.7% during the last ten years, its influence on vulnerability is only 5% of the total change. Under the fast growing energy import, effect of improving energy efficiency is insignificant. However the rebound of energy intensity from 2003 to 2005 shows its asymmetry effect on vulnerability, which means that increase of energy intensity will cause more change of vulnerability than decrease of it at the same range. Therefore strategy of energy saving is necessary from the view of vulnerability.

4.2. Compared with America

China has been a net importer of energy for just 12 years, shorter than most of the developed countries which have suffered crisis of energy import many times. Hence there are rich experience for us to study from them, especially America who is the largest country of energy consumption and import in the world. While the progress of structural change on China’s energy import/export during the last 12 years is very similar with America from 1949 to 2007, such as the increasing import of oil and peak of coal export.

Figure 3 shows the indicators of vulnerability as well as energy intensity of those two countries. We can see the same trend of increasing vulnerability and decreasing energy intensity both in China and America, as well as the peak of America’s vulnerability during the first two oil crisis. This section will compare the change of economic vulnerability to energy import be-
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tween them. Energy data and GDP used here are collected from EIA and BEA separately\(^\text{[30,31]}\), while the unit of energy is converted into btu, and unit of GDP is converted into 2000 constant dollars at purchasing power parity.

![Chart 3](image3.png)

**Fig.3.** America and China’s vulnerability indicators and energy intensity 1949/1991–2007

Change of America’s vulnerability indicator from 1953 to 2007 is decomposed by LMDI approach as shown in Fig.4, from which we can see the similar effects of each influence factor as in China, as well as the peak of vulnerability in the middle of 1970s mainly caused by declining oil production and oil crisis. Factor of energy dependence on import dominants the change of vulnerability in both countries, while increasing oil import takes about 2/3 to the total change. However effect of energy structural change in America is insignificant. Table 1 shows the results of decomposition in China and America, while effects of each kind of factor are calculated as the percentage of total vulnerability change.

![Chart 4](image4.png)

**Fig.4.** Decomposition of America’s vulnerability indicator 1953–2007
From this table we can find that effect of energy intensity on vulnerability in America is much larger than that of China, which indicates that improving energy efficiency is the key to ensuring economic security to energy import in the long term. Accumulated effect of energy intensity is -1642 btu / dollar in America from 1953 to 2007, takes 70.8% of the total vulnerability change, while the ratio of China is only 3.58% from 1996 to 2007. As a matter of fact, there was also little effect of energy intensity on vulnerability change in the first decade of America’s energy import as shown in Fig.4. Intensity effect of America from 1953 to 1970 is only 3.39 btu / dollar, 0.21% of the vulnerability change. After the second oil crisis, accumulated effect of energy intensity is as high as -1001 btu / dollar from 1980 to 2007, corresponding to the vulnerability change of 1137 btu / dollar. Hence the strategy of energy saving should be carried out in the long term.

Table 1 Comparison of decomposition on China and America’s vulnerability indicators

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Increase of Vulnerability (Btu/2000$)</th>
<th>Decomposition of vulnerability change (Total=100)</th>
<th>Effects by influence factors</th>
<th>Effects by energy type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Vulnerability</td>
<td>Import</td>
<td>Structure</td>
</tr>
<tr>
<td>China</td>
<td>1996-2007</td>
<td>824.63</td>
<td>110.69 -7.12 -3.57</td>
<td>40.73</td>
<td>64.51</td>
</tr>
<tr>
<td>America</td>
<td>1953-2007</td>
<td>2319.01</td>
<td>172.49 -1.69 -70.80</td>
<td>18.25</td>
<td>66.49</td>
</tr>
</tbody>
</table>

The structural change of America’s energy consumption is little the last twenty years, so is its effects on vulnerability. Ratio of oil, coal and natural gas consumption to total primary energy consumption of America in 2007 are 39%, 23% and 23%, while these numbers are 41%, 30% and 21% separately in 1954. As the main kind of import energy, the ratio of oil consumption in America reaches its peak at 47.6% in 1977 when the vulnerability also reaches its peak value, and the highest ratio of natural gas consumption is 32.4% in 1971. The consumption ratio of oil and natural gas both increase first and decrease afterwards, which results in the insignificant effect of energy structure.

Effect of increasing gas import in America during the last 50 years is 15% of the vulnerability change, closed to the effect of decreasing coal export. Although the ratio of natural gas consumption in America is 23% both in 1954 and 2007, the average speed of natural gas import is 7.8% per year from 1958 to 2007, while dependence rate of natural gas on import in America also increases from 0.9% to 16.5%. China’s natural gas consumption and production in 2007 is 67.3 and 69.3 billion cubic meter separately. According to China’s eleventh fifth-year plan of energy development, it will be a net importer of natural gas in 2010. Hence precaution should be taken against the vulnerability of natural gas.

4.3. International Experience

Decomposition analysis on China and America indicates that their economic vulnerability to
energy import are all dominated by the change of energy import. Fig.5 is the scatter diagram of vulnerability indicators and external dependence rate of energy supply in nearly 80 countries all over the world, which shows the positive relationship between two indicators. Cross-section data are collected from IEA where unit of GDP is 2000 dollar at purchasing power parity[32]. Red diamond in left bottom represents China whose dependence rate and economic vulnerability to energy import is in the low level of the world, and USA is the black square with vulnerability and import dependence three times larger than China.

The increase of economic vulnerability to energy import is not an inevitable trend, in many developed countries such as German, Italy and Spain, vulnerability indicators are declining under the growth of external energy dependence during the last 20 years, which means that effect of energy intensity is much larger than that of import. Hence improving energy efficiency is both necessary and validity in the long term for ensuring energy security.

![Fig.5. Dependence rate and economic vulnerability to energy import of main countries 2006](image)

5. Conclusions

In this paper, economic vulnerability to energy import is measured by the ratio of net energy import to GDP, and its change is decomposed into effects of energy import, energy structure and energy intensity employing LMDI approach, so as to study the relationship between energy import and economic security in China. Decomposition analysis of China indicates the dominant effect of energy import on increasing vulnerability from 1997 to 2007, as well as the insignificant effect of energy intensity and structure change. Cross-section data of main energy import countries also presents the positive relationship between external energy dependence and vulnerability. Hence following the increase of energy import in China, its economic vulnerability will rise inevitably.

Change of energy intensity and energy structure are the main way to restrain economic vulnerability to energy import, which had little impact in China during the last 10 years. Results of this study do not find remarkable effects of energy structure change on vulnerability.
in America either since structural change of them are all limited. However decomposition on America from 1954 to 2007 confirms the major influence of improving energy efficiency, which is 70.8% of the accumulated vulnerability change. In developed countries with high rate of external energy dependence rate such as German and Italy, vulnerability even decreases under the rise of energy import. Therefore strategy of energy saving is necessary and validity in the long term for China.

References:

Application of Disaggregated Logit Modeling with Stated Preference Methods in Park and Ride Behavior for Non-local Private Car Travelers in Big Events*

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Abstract: During big events, non-local private car travelers can be divided into two types which were returning in one day and in several days. In the paper it was demonstrated that those two kinds of travelers have distinct behavior on park and ride (P&R), due to their different traveling demand and behavior attributes. Focusing on the travelers returning in several days, the traveling attributes and requirements for P&R were analyzed. A P&R choice behavior disaggregated logit model was established and calibrated based on random utility theory. The model concludes three variables, which were travel time, parking fee and comprehensive attractiveness index for suburban satellite towns comparing to urban district. The results revealed that for travelers returning in several days the primary key point is increasing the attractiveness of suburban satellite towns.

Key words: travel behavior; big events; non-local private car travelers returning in several day; park and ride; disaggregated logit model; attractiveness index of suburban satellite towns

1. Introduction

In recent years many big events have been held in cities of China. Since the relationship between the transportation demands of big events and daily common operation of the city has to be deeply corresponded, urban transportation system is greatly challenged. One of the key points is the organization and parking problems of private car visitors. Park & Ride

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(hereinafter abbreviated as P&R) as one of the important measure for the questions has been put forward in big events transportation system planning. Quiet a large part of private car visitors are predicted from other cities and provinces, as well as they will stay in the hosted city for different days (Xiong Ping, 2006)[1]. For a example, during the 2010 Shanghai World Expo there will be 34,800–69,500 private car travelers (Pan Haixiao, 2005)[2] from Yangtze Delta Region in peak days, of which 70% will stay in Shanghai for several days (Xiong Ping, 2006)[1]. Therefore, the common P&R planning method for intraday commuters does not fit for non-local private car (hereinafter abbreviated as NLPC) travelers during big events. Based on their different remaining time, NLPC travelers in big events can be divided into two types: returning in one day and in several days. The research demonstrated that those two kinds of travelers have distinct behavior on P&R, due to their different traveling demand and behavior attributes (Xiong Ping, 2006)[1]. There has been some research on P&R behavior abroad. Verhoef (1995)[3] suggested that the travelers decision can be affected by adjusting parking policy. Lam (2001)[4] and Yang (2003)[5] carried on research respectively on P&R location decision-making and parking fee pricing from the angle of P&R behavior. Huang (2002)[6] and Tabuchi (1993)[7] studied the P&R behavior on flexible demand in peak hours. On the other hand there are relatively less research on P&R domestically, which mainly focused on P&R demand forecast, efficiency estimation (Pei Yulong, 2005; Chen Gang, 2005)[8,9] and so on, especially rare from the angle of behavior that including P&R behavior investigation and analysis methods etc. still remains primary stage (Qin Huanmei, 2005)[10]. There has no research both domestic and abroad for the question of P&R behavior of NLPC travelers returning in several days during big events.

On the background of the 2010 Shanghai World Expo, focusing on NLPC travelers returning in several days, the traveling attributes and requirements for P&R were analyzed in this paper. A P&R disaggregate logit model was established based on Random Utility Theory. The model was calibrated and analyzed based on P&R behavior Stated Preference (hereinafter abbreviated as SP) investigation. Correspondingly transportation suggestions were brought forward in the end.

2. Random Utility Function Formulation for P&R Behavior

When NLPC travelers arrive at hosted city during big events, they face two alternatives: $C_1$ and $C_2$, which represent respectively choosing P&R mode and keeping on driving private car. According to Random Utility Maximum Theory proposed by McFadden (1973)[11], each alternative holds certain utility. Individual $n$ will choose the alternative holding maximum utility. Utility function is linearly composed of two parts: nonrandom part $V_{in}$ and random part $\varepsilon_{in}$. $V_{in}$ is the determinate item calculated by observed factors $X_{ink}$, which consists of two parts: characteristic variables of both alternative and individual $n$. $\varepsilon_{in}$ is the random item which expresses unobservable factors and the bias caused by existing variables. Since $U_{in}$ includes a
random variable $\varepsilon_{in}$, $U_{in}$ itself is a random variable.

Suppose a NLPC traveler select alternative $C_i$ ($i=1, 2$) and the utility of $C_i$ is $U_{in}$. Then $U_{in}$ can be expressed as:

$$U_{in} = V_{in} + \varepsilon_{in}$$

(1)

And the probability $P_{in}$ for the traveler choosing alternative $C_i$ can be expressed as:

$$P_{in} = \text{Prob}\{\varepsilon_{jn} \leq \varepsilon_{in} + (V_{in} - V_{jn})\}$$

(2)

3. Journey Analysis for NLPC Traveler

For NLPC travelers returning in several days, there are two ultimate differences from NLPC travelers returning in one day and commuters. One is that the former has accommodation demand. The other is that the former will use cars traveling in the hosted city. Thus journey analysis of the NLPC travelers should be carried on from these two aspects.

**Hypothesis I** Travelers choose a downtown hotel during big events. The possible journey arrangement will be as follows:

(1) Firstly arrive at the hotel, and then go to EXPO. There will be three alternatives.
   (i) Choosing P&R mode. Transferring public transit to the hotel and giving up car driving entirely in the hosted city.
   (ii) Not choosing P&R mode. Driving a car to the hotel and then to EXPO.
   (iii) Not choosing P&R mode. Driving a car to the hotel and then using other transport mode to EXPO.

(2) Go to EXPO directly and return to the hotel after finishing visit. There will be two alternatives.
   (i) Choosing P&R mode. Transferring public transit to EXPO and giving up car driving entirely in the hosted city.
   (ii) Not choosing P&R mode. Going to EXPO and then return to the hotel.

**Hypothesis II** Travelers choose a hotel in suburban satellite towns during big events. And the P&R system is established combining with the suburban hotel. That is travelers can make convenient use of P&R system when choosing the hotel. The possible journey will be as follows:

(1) Firstly arrive at the hotel, and then go to EXPO. There will be two alternatives.
   (i) Choosing P&R mode. Transferring public transit to EXPO and return after finishing visit by transit.
   (ii) Not choosing P&R mode. Going to EXPO and return by a car.

(2) Go to EXPO directly and return to the hotel after finishing visit. There will be two alternatives.
   (i) Choosing P&R mode. Transferring public transit to EXPO and giving up car driving entirely in the hosted city.
(ii) Not choosing P&R mode. Driving a car to EXPO and then return to the hotel.

All the hypothetic alternatives above can be as shown in Figure 1. As for the alternative (i) under hypothesis choosing P&R mode, it means that travelers live in downtown area, leave cars in outer city (suppose the P&R system generally located in outer city). Since their activities mainly occur in urban district whereas they has to give up cars, thus the alternative (i) has hardly any possibilities. Only if the case of alternative (i) under hypothesis II, that travelers choose hotels in suburban satellite towns combining P&R systems makes adopting P&R mode to enter urban district possible. As to this alternative, firstly attractiveness of suburban satellite towns should be reinforced, then NLPC travelers may choose living in the hotel there and be guided to adopt park and ride mode, finally the objective of damming car volume and make it transferring public transit can be realized. The attractiveness of suburban satellite towns mainly depends on the cheap hotel expense, lower even free parking fee, nice scenery, tour resource, special food, tickets and information service etc.

Fig. 1 Trip chain for NLPC travelers returning in several days

4. Formulation of Logit Model for NLPC Travelers Returning in Several Days

The above analysis shows that the behavior mode of NLPC travelers returning in several days differs far from that returning in one day. The key factor influencing the former behavior is hotel location decision and the next place is level of service and relative price of transferred public transit, while on the contrary level of service and relative price of transferred public transit are key factors for the latter. Therefore the characteristic vector $X_{ink}$ of determinate item $V_{in}$ in utility function for these two kinds of travelers is different, and the corresponding utility function is distinct.

Aim at NLPC travelers returning in several days, a new index $A_i$ is created which is “attractiveness index of suburban satellite towns”. $A_i$ defines the ratio of attractiveness of suburban satellite towns to that of urban district. Here suburban satellite towns especially refers to those will newly develop as EXPO base and will have some advantages in hotel expense, parking fee,
nice scenery, natural tour resource, special food, tickets and information service and so on in contrast with urban district. If \( A_i \) is more than 1, it indicates suburban satellite towns are more attractive than that of urban district for NLPC travelers returning in several days, and which is true vice versa. Similarly the attractiveness of urban district is 1.

For the alternative \( C_1 \) above mentioned, the vector \( X_{ink} \) in its utility function are selected as follows: three characteristic variables of alternative itself, including \( A_i \), travel time of transferred public transit and parking fee of P&R system; and salary as a characteristic variable of individual. The determinate item \( V_{in} \) in utility function of \( C_1 \) can be expressed as:

\[
V_{1n} = \sum d_k X_{1nk} = d_0 d_1 A_{1n} d_2 F_{p1n} + d_3 T_{1n} + d_4 S_{1n}
\]  

(3)

Similarly the determinate item \( V_{in} \) in utility function of \( C_2 \) can be expressed as:

\[
V_{2n} = \sum d_k X_{2nk} = d_0 d_1 A_{2n} d_2 F_{p2n} + d_3 T_{2n}
\]  

(4)

where \( X_{ink} \) is a vector of observed variables of alternative \( i \) for individual \( n \), \( d_k \) is a corresponding coefficients vector which may vary over individuals but does not vary across alternatives or time, including \( d_0 \), \( d_1 \), \( d_2 \), \( d_3 \), \( d_4 \), \( A_{in} \) is the attractiveness index of suburban satellite towns for individual \( n \), \( F_{pin} \) is parking fee of alternative \( i \) for individual \( n \), \( T_{in} \) is travel time of alternative \( i \) for individual \( n \), \( S_{in} \) is the salary of individual \( n \). It can be summarized in the following Table 1.

<table>
<thead>
<tr>
<th></th>
<th>dummy variable</th>
<th>( A_{in} )</th>
<th>Parking fee</th>
<th>Travel time</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>P&amp;R mode(( C_1 ))</td>
<td>1</td>
<td>( A_{1n} )</td>
<td>( F_{p1n} )</td>
<td>( T_{1n} )</td>
<td>( S_{1n} )</td>
</tr>
<tr>
<td>Private car(( C_2 ))</td>
<td>0</td>
<td>( A_{2n} )</td>
<td>( F_{p2n} )</td>
<td>( T_{2n} )</td>
<td>0</td>
</tr>
<tr>
<td>Coefficients</td>
<td>( d_0 )</td>
<td>( d_1 )</td>
<td>( d_3 )</td>
<td>( d_2 )</td>
<td>( d_4 )</td>
</tr>
</tbody>
</table>

\( \varepsilon_{in} \) is an unobserved random term that captures the idiosyncratic effect of omitted variables during each choice occasion. \( \varepsilon_{in} \) is assumed to be independent of \( X_{ink} \) and \( d_k \). Here \( \varepsilon_{in} \) is assumed to be independently and identically Gumbel distributed across alternatives and individuals for each choice occasion. Its scale parameter is specified as \( \left( \frac{1}{\mu} \sum e^{\mu \eta_i}, \sigma \right) \) and its distributing density function is \( F(\varepsilon) = e^{-\mu(\varepsilon-\eta)} \exp[-e^{-\mu(\varepsilon-\eta)}] \).

After formulating determinate term and random term, the utility of alternatives can be obtained. According to logit model founded by McFadden, P&R behavior logit can be established. The probability \( P_{in} \) for NLPC travelers returning in several days choosing alternative \( C_1 \) (P&R mode) can be expressed as:

\[
P_{in} = \frac{e^{d_0 + d_1 A_{1n} + d_2 F_{p1n} + d_3 T_{1n} + d_4 S_{1n}}}{e^{d_0 + d_1 A_{1n} + d_2 F_{p1n} + d_3 T_{1n} + d_4 S_{1n}} + e^{d_1 A_{2n} + d_2 F_{p2n} + d_3 T_{2n}}}
\]  

(5)
5. Design and Practice of SP Investigation

SP investigation questionnaire is designed according to the model formula (5). Alternatives involve three variables: travel time, parking fee and $A_i$.

For the concision and convenience of question design and practice, travel time, parking fee and $A_i$ of alternative $C_2$(private car mode) is initialized as 1 and calculated from the point of P&R system. The three variables of alternative $C_1$(P&R mode) is figured as increased or decreased percentage compared with those of $C_2$ as follows concretely.

- Set three levels for increased percentage of travel time after choosing P&R system: 0, 15%, 30%.
- Set three levels for increased percentage of parking fee after choosing P&R system: 25%, 50%, 100%.
- Set two levels for $A_{1n}$. $<1$(suppose it is equal to 0.8); $>1$(suppose it is equal to 1.2).

Interviewees are required to give choice among the following three options under the above supposed conditions: (a) You will not choose the hotel in suburban satellite towns'; (b) You will choose the hotel in suburban satellite towns but drive car to EXPO; (c) You will choose the hotel in suburban satellite towns and utilize P&R system to EXPO; SP investigation was carried out from 8:00 to 18:00 in Labor’s Day at Fengjing and Jiaxing service district along Huhang expressway. Interviewees were selected from those private car travelers from Zhejing and Jiangsu province to Shanghai and returning in several days. Investigation method was that investigator inquired interviewees face to face. Since there was impossible to get sample matrix, nonrandom sampling was implemented. Finally 3636 samples were obtained in the SP investigation.

6. Calibration and Analysis of the Logit Model

Based on the SP investigation data, coefficients $d_k$ of the logit model can be calibrated through Maximum Likelihood Estimate. Likelihood function $L^* = \prod_{n=1}^{N} P_{2n}^{d_{2n}} P_{2n}^{d_{2n}}$ was formed based on the model equation (5). Then the logarithm of $L^*$ was operated and the log-likelihood function

$$L = \sum_{i=1}^{N} \left[ \delta_{1n} \ln \left( \frac{1}{1 + e^{-\theta'(X_{1n} - X_{2n})}} \right) + \delta_{2n} \ln \left( \frac{e^{-\theta'(X_{1n} - X_{2n})}}{1 + e^{-\theta'(X_{1n} - X_{2n})}} \right) \right]$$

was gotten. Then the partial derivative of $d_k$ was operated and set to zero, thus the value of $d_k$ can be gained. The calculation was achieved through a Matlab program based on Newton-Raphson algorithm.

Because the salary inquiry results were unauthentic due to the private characteristic, the $t$ test of the value of coefficient $d_4$ for salary was unqualified after the first round of the calculation. And the $t$ test of the value of coefficient $d_4$ for salary was unqualified too. Therefore those two items of salary and travel time were excluded from the characteristic vector $X_{ink}$ and the model
was recalculate till all coefficients passing $t$ test. The final calibration result of the model was:

$$P_{1n} = \frac{1}{1 + e^{1.9890 - 1.3233\Delta F_p - 4.4363\Delta A}}$$

(6)

where $\Delta p$ is absolute value of parking fee saving for P&R system compared with parking near EXPO entrance when the parking fee near EXPO entrance is initialized as 1. $\Delta A$ is the value of attractiveness of suburban satellite towns minus that of urban district when the latter is initializes as 1.

The calibration result of the model showed that the most important factor is the attractiveness of suburban satellite towns for NLPC travelers returning in several days. The coefficient of $\Delta A(4.4363)$ is several times of that of $\Delta F_p(1.3233)$, and the impact of travel time was negligible. Whether the attractive of suburban satellite towns is more than that of urban district or not had a great influence on the outcome of P&R choice as shown in Table 2. The attractiveness of suburban satellite towns to travelers as EXPO base depends on many factors, including development in 2010, infrastructure, resources, environments, supporting services and so on, and also is influenced by requirements and value judgment of individuals. Thus the index $\Delta A$ itself needs in-depth study. The paper only study the influences on P&R behavior by $\Delta A$ under two types of situations, greater than 1 and less than 1 respectively.

Table 2 revealed that when the attractiveness of suburban satellite towns is low $P_{1n}$ is close to zero, and impact of parking fee saving no $P_{1n}$ is almost neglected. On the other hand, when the attractiveness of suburban satellite towns surpasses that of urban district, $P_{1n}$ can come to 60% and diverse largely with the changes of $\Delta F_p$.

<table>
<thead>
<tr>
<th>$\Delta F_p$</th>
<th>100%</th>
<th>100%</th>
<th>100%</th>
<th>100%</th>
<th>100%</th>
<th>100%</th>
<th>50%</th>
<th>50%</th>
<th>50%</th>
<th>50%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta A$</td>
<td>-0.5</td>
<td>-0.3</td>
<td>-0.1</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>-0.5</td>
<td>-0.3</td>
<td>-0.1</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>$P_{1n}$</td>
<td>3.3%</td>
<td>9.1%</td>
<td>23.0%</td>
<td>33.9%</td>
<td>47.0%</td>
<td>60.4%</td>
<td>1.7%</td>
<td>4.9%</td>
<td>13.3%</td>
<td>21.0%</td>
<td>31.4%</td>
</tr>
</tbody>
</table>

From the above analysis, it confirmed again that, in order to attract NLPC travelers returning in several days using P&R system, the primary key is to enhance the attractiveness of suburban satellite towns. As long as travelers choose living in suburban satellite towns, then it is possible for them to adopt P&R mode. P&R planning can be carried on accordingly including policy and level of service of transferred public transit etc. calculated from the model.

7. Summary and Conclusions

Two kinds of NLPC travelers in big events, which were returning in one day and in several days, have distinct behavior on park and ride (P&R), due to their different traveling demand and behavior attributes. Focusing on the NLPC travelers returning in several days, the traveling attributes and requirements for P&R were analyzed. A P&R choice behavior disaggregated logit
model was established and calibrated based on random utility theory. The model concludes three variables, which were travel time of P&R transit, diversity of parking fee and comprehensive attractiveness index for suburban satellite towns comparing to urban district. The results revealed that besides improving public transit attractiveness and providing information service, the planning and policy of P&R for NLPC travelers returning in several days should concern more on the requirements of the whole trip chain. The key point is increasing the attractiveness of suburban satellite towns. The suggestion of establishing P&R system combining hotels at suburb has been put forward.

References:


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