

## Introduction to Grey System Theory

Deng Julong

Department of Automation, Huazhong University of Science and Technology, Wuhan,  
People's Republic of China

Received October 1988

### Outline

Grey System theory was initiated in 1982 [7]. As far as information is concerned, the systems which lack information, such as structure message, operation mechanism and behaviour document, are referred to as Grey Systems. For example, the human body, agriculture, economy, etc., are Grey Systems.

Usually, on the grounds of existing grey relations, grey elements, grey numbers (denoted by  $\otimes$ ) one can identify which Grey System is, where "grey" means poor, incomplete, uncertain, etc. The goal of Grey System and its applications is to bridge the gap existing between social science and natural science. Thus, one can say that the Grey System theory is interdisciplinary, cutting across a variety of specialized fields, and it is evident that Grey System theory stands the test of time since 1982.

As the case stands, the development of the Grey System—as well as theoretical topic—is coupled with clear applications of the theory in assorted fields.

The concept of the Grey System, in its theory and successful application, is now well known in China. The application fields of the Grey System involve agriculture [23, 77-81, 91], ecology [59], economy [61, 102, 103, 104], meteorology [58, 74, 91], medicine [55, 89], history [63, 64], geography [1], industry [6], earthquake [73, 87, 88], geology [76, 119], hydrology [98, 112], irrigation strategy [26], military affairs, sports [116], traffic [67], management [30, 97, 105], material science [82, 83], environment [108], biological protection [69, 70], judicial system [100], etc.

Projects which have been successfully completed with the Grey System theory and its applications are as follows:

1. Regional economic planning for several provinces in China;
2. To forecast yields of grain for some provinces in China;

© 1989 Sci-Tech Information Services and China Petroleum Industry Press

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the co-publisher, Sci-Tech Information Services, P.O.Box 991, Windsor, Berks SL4 4FR, U.K.

*Special regulations for readers in the USA.* This journal has been registered with the Copyright Clearance Center, Inc. Consent is given for copying of articles for personal or internal use, or for the personal or internal use of specific clients. This consent is given on the condition that the copier pay through the Center the per-copy fee stated in the code on the first page of each article for copying beyond that permitted by Sections 107 or 108 of the US Copyright Law. The appropriate fee should be forwarded, quoting the code number at the end of this paragraph, to the Copyright Clearance Centre, 21 Congress Street, Salem, MA 01970, USA. If no code appears in an article, the author has not given broad consent to copy and permission to copy must be obtained directly from the author. This consent does not extend to other kinds of copying, such as for general distribution, resale, advertising and promotion purposes, or for creating new collective works. Special written permission must be obtained from the publisher for such copying. 0957-3720 / 89 / \$ 03.50

*Special regulations for authors.* Upon acceptance of an article by the journal, the author(s) will be asked to transfer copyright of the article to the publisher. The transfer will ensure the widest possible dissemination of information.

*Note.* No responsibility is assumed by the Publishers for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions or ideas contained in the material herein.

Although all advertising material is expected to conform to ethical standards, inclusion in this publication does not constitute a guarantee or endorsement of the quality or value of such product or of the claims made of it by its manufacturer.

Managing Editor: Zhang Jixian, Director of International Dept., CPIP  
Typesetting: Liu Xiaohua, Zhang Hong  
Printed in China by The Printing Factory of Petroleum Industry Press

3. To analyse agricultural economy in China;
4. To make satisfactory planning of irrigation (watershed of the People's Victory Channel in Henan Province);
5. To build models available for biological protection;
6. To control the water level for boilers by grey prediction control;
7. To estimate the economic effect;
8. To build a diagnosis model available for medicine;
9. To forecast weather.

As far as properties of implement fields are concerned, it can be seen that the greatest part of them can be referred to as abstract systems, which are short of physical prototypes, according to the terminology used throughout the Grey System theory. They can be called latent Grey Systems.

The aims of Grey System theory are to provide theory, techniques, notions and ideas for resolving (analysing) latent and intricate systems, for example:

1. To establish a non-function model instead of regressive analysing;
2. To define and constitute Grey Process replacing the stochastic process and to find the real time techniques instead of statistical model to deal with the grey process, in order to obtain an approach to modelling with few data, avoiding searching for data in quantities;
3. To turn the disorderly raw data into a more regular series by grey generating techniques for the benefit of modelling instead of modelling with original data;
4. To build a differential model—so called grey model (GM)—by using the least 4 data to replace difference modelling in vast quantities of data;
5. To develop a novel family of grey forecasting methods instead of time series and regressive methods—all of these may be referred to as an approach to deal with grey processes;
6. To develop innovational techniques and concepts for decision making—so called grey decision making;
7. To develop novel control techniques. e. g., the grey forecasting control replacing classical control which is referred to as afterward control, also relational control, generating control and programming control;
8. To study the mechanism theory, including grey sequence theory and grey structure theory;
9. To study feeling and emotion functions and fields with whitening functions;
10. Based on grey relations, grey elements and grey numbers to be used to study grey mathematics instead of classical mathematics study.

The essential contents and topics of Grey System theory encompass the following areas:

grey relational space  
grey generating space  
grey forecasting

grey decision making  
grey control  
grey mathematics  
grey theory

Why have so many researchers in the fields of software science utilized Grey System theory, and why has there been dialogue between them and the Grey System theory? At least two reasons immediately spring to mind. First, the software researchers tend to be methodologically accepting, when they are grey mathematically efficient and sure of being grey methodologically innovative; second, some of the element techniques and concepts of Grey System theory have been implemented and investigated by a variety of areas and even incorporated in some theoretical schema, such as grey market and economic programming.

### Grey Relational Space

The grey relational space (GRS) [11, 18] is one that describes the posture relationships between one main factor and all the other factors in a given system. A GRS is a binary set denoted by  $(X, \Gamma)$  where  $X$  is a collection composed of sequences  $x_i$  to be compared and reference sequence  $x_0$ ,  $\Gamma$  is a map set called grey relational map set,  $\gamma \in \Gamma$  is an appointed relational map in GRS. Assume that

$$\gamma(x_0(k), x_i(k))$$

is an image at point  $k$  from the series to real number with map  $\gamma$  and  $\gamma(x_0, x_i)$  is an image at all points with  $k = 1, 2, 3, \dots, n$ , where

$$x_0 = (x_0(1), \dots, x_0(n))$$

$$x_i = (x_i(1), \dots, x_i(n))$$

Let  $\gamma(x_0, x_i)$  satisfy that

$$\gamma(x_0, x_i) = \frac{1}{n} \sum_{k=1}^n \gamma(x_0(k), x_i(k))$$

then  $\gamma(x_0(k), x_i(k))$  is said to be a grey relational coefficient at the point  $k$  and  $\gamma(x_0, x_i)$  to be a grey relational grade, iff  $\Gamma$  satisfies the following axioms [18]:

## (A1) (Norm Interval)

$$\begin{aligned} \gamma(x_0(k), x_i(k)) &\in (0, 1], \forall k \\ \gamma(x_0(k), x_i(k)) &= 1, \text{ iff } x_0(k) = x_i(k), \forall k \\ \gamma(x_0(k), x_i(k)) &= 0, \text{ iff } x_0 \in \varphi, x_i \in \varphi \end{aligned}$$

where  $\varphi$  is an empty set.

## (A2) (Duality Symmetric)

$$\begin{aligned} \gamma(x_0(k), x_i(k)) &= \gamma(x_i(k), x_0(k)), \text{ iff} \\ \mathbf{X} &= \{x_0, x_i\} \end{aligned}$$

## (A3) (Wholeness)

$$\gamma(x_0(k), x_i(k)) \neq \gamma(x_i(k), x_0(k)),$$

almost always, iff

$$\mathbf{X} = \{x_j | j = 0, 1, \dots, n, n > 2\}$$

## (A4) (Approachability)

$\gamma(x_0(k), x_i(k))$  decreases along with  $\Delta(k)$  increasing, where

$$\Delta(k) = [(x_0(k) - x_i(k))^2]^{\frac{1}{2}}$$

Now, we have a following expression for  $\gamma(x_0(k), x_i(k))$  which satisfies all of the mentioned axioms, that which

$$\gamma(x_0(k), x_i(k)) = \frac{\min_i \min_k |x_0(k) - x_i(k)| + \zeta \max_i \max_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \zeta \max_i \max_k |x_0(k) - x_i(k)|}$$

where  $\zeta \in (0, 1)$  is of the distinguished coefficient [53].

**Remark.** The norm interval  $(0,1]$  which means  $\forall \gamma(x_0(k), x_i(k)) \in (0,1] \forall k$  was first developed by Deng (1985)[18], while the norm interval  $[a, b]$ ,  $\gamma(x_0(k), x_i(k)) \in [a, b]$  where  $a, b$  are arbitrary real numbers was developed by Guo (1985)[53]. The concept of grey relational grade of interval was proposed by Wang (1985)[94].

A variety of successful applications for GRS have been exploited by many scientists in different areas, such as material science [83], biological analysis [84], seismological research [87], judicial system [100], economic analysis [117], and geology [76].

## Grey Generating Space

In the viewpoint of Grey System theory, the concept and generating techniques are important ideas by which the disorderly raw data can be turned to a regular series for the benefit of grey modelling, transferred to a dimensionless series in order to obtain an appropriate fundamental for grey analysing and changed into a unidirectional series for decision making.

Let  $x, y, \psi$  be the series and  $x \in \mathbf{X}, y \in \mathbf{Y}, \psi \in \Phi$ , where  $\mathbf{X}, \mathbf{Y}, \Phi$  are collections of dimension  $n$ . If

$$\begin{aligned} x &= \sum_{k=1}^n y(k) \psi_k \\ y(k) &\in y \in \mathbf{Y}, \psi_k \in \Phi, x \in \mathbf{X} \end{aligned}$$

then  $\Phi$  is a basis space,  $\psi_k$  a basis of  $\Phi$ ,  $x$  an image of  $y$  and  $\mathbf{X}$  an image space spanned by the basis  $\psi_k$  and the coordinates  $\mathbf{Y}$ .

Hence

$$(\mathbf{X}; \Phi / \mathbf{Y})$$

is a generating space [40].

Suppose that

$$\begin{aligned} x^{(0)} &= (x^{(0)}(1), \dots, x^{(0)}(n)) \\ x^{(1)} &= (x^{(1)}(1), \dots, x^{(1)}(n)) \\ x^{(1)}(k) &= \sum_{m=1}^k x^{(0)}(m) \end{aligned}$$

and that

$$\begin{aligned} x^{(0)} &\in \mathbf{Y}, x^{(1)} \in \mathbf{X}, \psi_k^{(1)} \in \Phi \\ \psi_k^{(0)} &= (0, \dots, 0, \underset{\substack{k\text{th} \\ \text{column}}}{1}, 0, \dots, 0) \\ \psi_k^{(1)} &= (0, \dots, 0, \underset{\substack{k\text{th} \\ \text{column}}}{1}, \dots, 1) \end{aligned}$$

$$\psi_k^{(1)}(m) \in \psi_k^{(1)}$$

$$\psi_k^{(1)}(m) = \sum_{\sigma=1}^m \psi_k^{(0)}(\sigma)$$

Then

$$x^{(1)} = \sum_{k=1}^n x^{(0)}(k) \psi_k^{(1)}$$

By AGO the above generating operation is denoted, i.e.

$$\text{AGO } x^{(0)} = x^{(1)}$$

$$x^{(1)}(k) = \sum_{m=1}^k x^{(0)}(m)$$

thus AGO  $x^{(0)}$  is said to be an Accumulated Generating Operation which is vital to grey modelling. Similarly, the  $j$ th order AGO is as follows:

$$x^{(i)}(k) = \sum_{m=1}^k x^{(i-1)}(m), \quad i = 1, 2, \dots$$

and that

$$x^{(i)} = \sum_{k=1}^n x^{(i-1)}(k) \psi_k^{(1)}$$

Moreover, some theorems concerning the  $j$ th order AGO of  $x^{(0)}$  have been established [40].

Assuming that

$$\alpha^{(0)}(x^{(i)}(k)) = x^{(i)}(k)$$

$$\alpha^{(1)}(x^{(r)}(k)) = \alpha^{(0)}(x^{(r)}(k)) - \alpha^{(0)}(x^{(r)}(k-1))$$

$$\vdots$$

$$\alpha^{(i)}(x^{(r)}(k)) = \alpha^{(i-1)}(x^{(r)}(k)) - \alpha^{(i-1)}(x^{(r)}(k-1))$$

$$r = 1, 2, \dots$$

then we have

$$\alpha^{(1)}(x^{(1)}) = \sum_{k=1}^n x^{(1)}(k) \psi_k^{(1)}$$

$$\psi_k^{(1)} = (0, \dots, 0, 1, \underset{\substack{\text{kth} \\ \text{column}}}{-1}, 0, \dots, 0)$$

$$\psi_k^{(1)} \in \Phi, x^{(1)}(k) \in X, \alpha^{(1)}(x^{(1)}) \in Y, \alpha^{(1)}(x^{(1)}) \in X$$

By IAGO the above-mentioned generating operation is denoted, such that

$$\text{IAGO } x^{(1)} = x^{(0)} = \alpha^{(1)}(x^{(1)})$$

Thus IAGO  $x^{(1)}$  is said to be an Inverse Accumulated Generating Operation.

Summing up the results of grey generating operations, we have

1. (AGO):  $x^{(1)}(k) = \sum_{m=1}^k x^{(0)}(m)$
2. (IAGO):  $\alpha^{(i)}(x^{(i)}(k)) = \alpha^{(i-1)}(x^{(i)}(k)) - \alpha^{(i-1)}(x^{(i)}(k-1))$
3. (MEAN):  $\bar{z}^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$
4. (P):  $\bar{x} = (\bar{x}(1), \dots, \bar{x}(n)), \quad x = (x(1), \dots, x(n)),$   
 $x(k) = \bar{x}(k) / x(p), \quad x(p) = \frac{1}{n} \sum_{m=1}^n \bar{x}(m)$
5. (INIT):  $\bar{x} = (x(1), \dots, \bar{x}(n)), \quad x = (x(1), \dots, x(n)),$   
 $x(k) = \bar{x}(k) / \bar{x}(1)$

By AGO, the following important results can be obtained; the non-negative, smooth, discrete function can be transferred into a series, extended according to an approximate exponential law [36,41] which is called the grey exponential law and by which a reform to establish a suitable foundation in building a differential model is said to be completed.

## Grey Modeling

On the subject of differential modeling, T.C. Hsia follows that, for biological phenomena, economy and biological medicine, one had better build a differential model, but this has been impossible in the past. In Grey System theory a dynamic model with a group of differential equations is developed [15, 20, 42, 44], which is called grey differential model (GM). To do this Deng (1985)[20] inferred:

1. A stochastic process whose amplitudes vary with time is referred to as a grey process;
2. The grey modelling is based on the generating series rather than on the raw one;
3. The grey derivative, parallel shooting and grey differential equation are

defined and proposed in order to build a GM:

4. To build a GM model, only a few data (as few as 4) are needed to distinguish it.

Let  $x^{(0)}$  be a raw series,  $x^{(1)} = \text{AGO}x^{(0)}$ , then the following equation

$$x^{(0)}(k) + az^{(1)}(k) = b, \quad k = 1, 2, \dots, n, \dots$$

is a grey differential model, called GM(1,1) as it includes only one variable  $x^{(0)}$ , where

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), \quad k = 1, 2, \dots, n, \dots$$

$a, b$ , are the coefficients; in Grey System theory terms,  $a$  is said to be a developing coefficient and  $b$  the grey input,  $x^{(0)}(k)$  is a grey derivative which maximizes the information density for a given series to be modelled.

According to the least square method, we have

$$\hat{\mathbf{a}} = \begin{bmatrix} a \\ b \end{bmatrix} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{y}_N$$

there

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ \dots & \dots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad \mathbf{y}_N = \begin{bmatrix} x^{(0)}(2) \\ \dots \\ x^{(0)}(n) \end{bmatrix}$$

Here  $\mathbf{B}$  is called a data matrix.

By regarding the following equation

$$\frac{dx^{(1)}}{dt} + a \otimes(x^{(0)}) = b$$

as a shadow for  $x^{(0)}(k) + az^{(1)}(k) = b$

We have

$$\begin{aligned} \mathcal{X}^{(1)} &= \otimes(x^{(1)}) = \{x^{(1)} | \forall x^{(1)} \in \mathcal{X}^{(1)}\} \\ &= \left\{ \tilde{\otimes}(x^{(1)}) | \tilde{\otimes}(x^{(1)}) \begin{cases} \rightarrow x^{(1)}(t + \Delta t) / \Delta t \\ \rightarrow x^{(1)}(t - \Delta t) / \Delta t \end{cases} \right\} \end{aligned}$$

where  $\otimes(x^{(1)})$  is said to be a background grey number for

$$dx^{(1)} / dt, \tilde{\otimes}(x^{(1)})$$

is the whitening value of grey number  $\otimes(x^{(1)})$  and

$$\tilde{\otimes}(x^{(1)}) \begin{cases} \rightarrow x^{(1)}(t + \Delta t) / \Delta t \\ \rightarrow x^{(1)}(t - \Delta t) / \Delta t \end{cases}$$

means the parallel morphism or shooting from  $\tilde{\otimes}(x^{(1)})$  to the components of  $dx^{(1)} / dt$  [20, 35], then the response equation for GM(1,1) are as follows

$$\begin{aligned} \hat{x}^{(1)}(k+1) &= \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \\ \hat{x}^{(0)}(k+1) &= \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k). \end{aligned}$$

where  $\hat{x}^{(1)}(k)$ ,  $\hat{x}^{(0)}(k)$  means calculating values of  $x^{(1)}$  and  $x^{(0)}$  at point  $k$ .

A grey differential equation having  $N$  variables is called GM(1, $N$ ), whose expression can be written as follows:

$$x_1^{(0)}(k) + az_1^{(1)}(k) = \sum_{i=2}^N b_i x_i^{(1)}(k), \quad k = 1, 2, \dots$$

where  $b_i$  is said to be an  $i$ th influence coefficient, which means that  $x_i$  exercises influence on  $x_1$  (the behaviour variable). Then we have

$$\hat{\mathbf{a}} = [a, b_2, \dots, b_N]^T$$

By means of least square method, we have

$$\hat{\mathbf{a}} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{y}_N$$

$$\mathbf{B} = \begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) & \dots & x_N^{(1)}(2) \\ \dots & \dots & \dots & \dots \\ -z_1^{(1)}(n) & x_2^{(1)}(n) & \dots & x_N^{(1)}(n) \end{bmatrix}, \quad \mathbf{y}_N = \begin{bmatrix} x_1^{(0)}(2) \\ \vdots \\ x_1^{(0)}(n) \end{bmatrix}$$

Model GM(1,1) plays an important role in grey forecasting, grey programming and grey control. Model GM(1, $N$ ) has laid an important foundation for regional economic programming and grey multivariable control [42, 102].

### Grey Forecasting

The subjects of grey forecasting [21, 43] include:

- series forecasting
- calamities forecasting
- season calamities forecasting
- topological forecasting
- systematic forecasting

As all of these are based on GM(1,1), we call them grey forecasting, whose objective is to unify the field and to bridge the gap between grey process theory and practice. Our intention is to make forecasting useful for decision and policy makers who need future predictions.

### Grey Series Forecasting

Grey series prediction is a common technique available for forecasting—which is referred to essential prediction—and direct use of GM(1,1) will enable us to know where and how the events to be forecasted would appear.

Let  $x^{(0)}$  be a raw series, AGO  $x^{(0)} = x^{(1)}$ ,  $\hat{x}^{(0)}(\xi)$  the prediction value at point  $\xi$ , then the series prediction process can be written as

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO}: x^{(0)} \rightarrow \hat{x}^{(0)}(\xi)$$

$$x^{(0)} = (x^{(0)}(1), \dots, x^{(0)}(n)), \quad \xi > n,$$

where IAGO and AGO are maps which transfer  $\hat{x}^{(1)}$  to  $\hat{x}^{(0)}$  and  $x^{(0)}$  to  $x^{(1)}$  respectively, and GM implies the grey modelling for GM(1,1) by  $x^{(1)}$ . For a series prediction, a variety of successful instances have been completed in different areas, such as grain yields prediction [62, 79, 114], economic prediction [108, 92, 102], material science [82], epidemiology [89], electrical power [75], traffic [67], technological progress [85], seismology [88], judicial systems [100], engineering [120] and agriculture [117, 118].

### Calamities Prediction

While the amplitudes of events to be forecasted refer to an appointed interval called “grey number of pests”, then the predictions to know when the event will occur belong to calamities prediction [21, 43].

Let  $x^{(0)}$  be a raw series

$$x^{(0)} = (x^{(0)}(1), \dots, x^{(0)}(n))$$

$\otimes_L, \otimes_U$  represent grey numbers of pests such that

$$\otimes_L = \{x^{(0)}(k) | x^{(0)}(k) \in x^{(0)}, x^{(0)}(k) \leq \zeta\}$$

$$\otimes_U = \{x^{(0)}(k) | x^{(0)}(k) \in x^{(0)}, x^{(0)}(k) \geq \zeta\}$$

where  $\zeta$  is said to be a boundary value of pests and  $\otimes_L$  is a grey number of low pests which means that the pest value is less than  $\zeta$ ;  $\otimes_U$  is a grey number of upper pests in which the pest value is greater than  $\zeta$ . If we have a subseries  $x_{\zeta}^{(0)}$  of  $x^{(0)}$  such that:

$$x_{\zeta}^{(0)} = (x^{(0)}(1_{\zeta}), \dots, x^{(0)}(n_{\zeta}))$$

$$x^{(0)}(m_{\zeta}) \in \otimes_L \cap x^{(0)}, \quad m = 1, 2, \dots, n$$

or

$$x^{(0)}(m_{\zeta}) \in \otimes_U \cap x^{(0)}, \quad m = 1, 2, \dots, n$$

then

$$\otimes_L = \{x^{(0)}(m_{\zeta}) | x^{(0)}(m_{\zeta}) \in x^{(0)}, x^{(0)}(m_{\zeta}) \leq \zeta\}$$

$$\otimes_U = \{x^{(0)}(m_{\zeta}) | x^{(0)}(m_{\zeta}) \in x^{(0)}, x^{(0)}(m_{\zeta}) \geq \zeta\}$$

Suppose that  $p_{\zeta}, p_h$  and  $p_g$  are maps such that

$$p_{\zeta}: x^{(0)} \rightarrow x_{\zeta}^{(0)}$$

$$p_h: x^{(0)}(k) \rightarrow k$$

$$p_h: x^{(0)}(m_{\zeta}) \rightarrow m_{\zeta}$$

$$p_g: k_{\zeta} \rightarrow k$$

then the calamity prediction can be written as follows

$$p_g \cdot p_h \cdot p_{\zeta}: x^{(0)} \rightarrow (p_g(1_{\zeta}), p_g(2_{\zeta}), \dots, p_g(n_{\zeta}))$$

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO}: (p_g(1_{\zeta}), \dots, p_g(n_{\zeta})) \rightarrow (t_{\zeta})$$

where  $t_{\zeta}$  is a time to be forecasted when pests appear and  $(t_{\zeta})$  is a series of  $t_{\zeta}$ .

A variety of instances of pest predictions become effective in different areas, such as meteorology [115, 58] and agriculture [78].

**Seasonal Calamity Prediction**

The pests to be forecasted which happen in a certain span of time of a year are referred to as a seasonal calamity. It is obvious since an assignment to forecast the seasonal pests is similar to the above-stated prediction—namely, the calamity prediction—but their procedures are different [21, 43].

Let  $h(k)$  be a pest time (date) located in a special span of a year: its scalar quantity refers to a relative value regarding the minimum value among the samples of pest date, and  $h$  is the series of  $h(k)$

$$h = (h(1), \dots, h(n))$$

$h_k$  is the straight line given by

$$\begin{aligned} h_k &= m_k t + b \\ h_k &\in [\min(h(k-1), h(k)), \max(h(k-1), h(k))], \forall k \\ t &\in [k-1, k] \\ m_k &= h(k) - h(k-1) \\ b_k &= kh(k-1) - (k-1)h(k) \end{aligned}$$

where  $h_k$  is a straight line to link  $h(k-1)$  with  $h(k)$ , then the collection

$$Y = \{h_k | k = 1, 2, \dots\}$$

is a line graph called the seasonal pest graph.

Suppose  $d_i = \text{const}$  which means a fixed date of pest and assume that  $d_i$  cuts across  $Y$  at points 1, 2,  $\dots$ ,  $n$  and it is said to be a contour line in the pest graph. Let  $p_i(k)$  be the  $k$ th span from point 1 to  $k$ , then

$$p_i = (p_i(1), p_i(2), \dots, p_i(n)), \quad d_i \Rightarrow p_i$$

is a series for modelling, iff  $p_i$  is non-empty, the potency of  $p_i$  is more than 4 and

$$\rho(\xi) = \frac{p_i(\xi)}{\sum_{m=1}^n p_i(m)} \leq \varepsilon < 1, \quad \varepsilon \in [0, 1]$$

The process of seasonal pest prediction thus can be written as

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO } p_i \Rightarrow \hat{p}_i^{(0)}(\xi) \\ \xi > n_i$$

where  $\hat{p}_i^{(0)}(\xi)$  is a year to be forecasted in which the seasonal pest would occur at date  $d_i$ .

Instances would be to forecast the date that the first frost would occur in Shanxi Province [91] and the date that the seasonal wind would appear in Hunan and Guangxi Provinces.

**Topological Prediction**

According to the above-mentioned procedures and techniques for seasonal pest prediction, forecasting the future form of a given curve is called topological prediction [21, 43].

Let us suppose that  $Y$  is a graph concerning a given curve such that

$$Y = \{h_k | k = 1, 2, \dots\}$$

$\{d_i\}$  represents a subset of points belonging to a set of points which may be located at a straight line to the given or another curve. Thus:

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO } \{p_i\} \Rightarrow \{\hat{p}_i^{(0)}(\xi)\}, \quad \{d_i\} \Rightarrow \{p_i\}$$

is a topological prediction model referred to  $\{d_i\}$  and  $\{\hat{p}_i^{(0)}(\xi)\}$  is a future curve to be forecasted. In order to analyse and to deal with a grey process whose curves are not smooth, the topological prediction techniques may be useful.

**Systematic Prediction**

To forecast a variety of different variables as a whole, referred to a given system, it is said to be a systematic prediction. If the assumption is made that the variables to be forecasted together are subordinated to, and modelled with, GM(1,1) and the results for every variable can be obtained recursively one by one, then such a forecasting is said to be a grey systematic prediction.

Let  $x_i$  ( $i=1, 2, \dots, n$ ) be the behaviour series referred to a given system, then the whole  $x_i$  can be forecasted by means of a grey systematic prediction, if the following relationships between variables  $x_i$ ,  $i=1, 2, \dots, n$  hold,

$$\begin{aligned} \frac{dx_1^{(1)}}{dt} + a_{11}x_1^{(1)} &= a_{12}x_2^{(1)} + \dots + a_{1n}x_n^{(1)} \\ \dots \\ \frac{dx_{n-1}^{(1)}}{dt} + a_{n-1,n-1}x_{n-1}^{(1)} &= a_{n-1,n}x_n^{(1)} \\ \frac{dx_n^{(1)}}{dt} + a_{n,n}x_n^{(1)} &= b \end{aligned}$$

where  $x_i^{(1)}$  ( $i=1, 2, \dots, n$ ) are the background grey numbers concerned with derivatives

$$\frac{dx_i^{(1)}}{dt}, \quad i=1, 2, \dots, n$$

Let us denote the procedure to model GM(1,1)

$$\frac{dx_n^{(1)}}{dt} + a_{nn}x_n^{(1)} = b$$

by  $GM_n$  and by  $gm_n$  the result or the model of it. As a function of  $gm_n$ , consider the  $n-1$ th grey differential equation, as follows:

$$\begin{aligned} \frac{dx_{n-1}^{(1)}}{dt} + a_{n-1,n-1}x_{n-1}^{(1)} &= a_{n-1,n}x_n^{(1)} \\ \Rightarrow \frac{dx_{n-1}^{(1)}}{dt} + a_{n-1,n-1}x_{n-1}^{(1)} &= a_{n-1,n}f(gm_n) \end{aligned}$$

Thus we can briefly denote it with

$$GM_{n-1} = (gm_{n-1}, gm_n) = f(gm_{n-1}, gm_n)$$

where  $GM_{n-1}$  implies the procedure to model  $n-1$ th relationship and  $(gm_{n-1}, gm_n)$  is the result from  $GM_{n-1}$  which involves models  $gm_n$  and  $gm_{n-1}$ . Hence, the grey systematic prediction can be presented briefly with the following representations:

$$\begin{aligned} GM_n \\ GM_{n-1} &= (gm_{n-1}, gm_n) \\ \dots \end{aligned}$$

$$GM_1 = (gm_1, gm_2, \dots, gm_n)$$

It is evident that to model a variety of variables together by the grey systematic prediction which is obtained directly from a variety of models, GM(1,1), the complex procedures which are necessary to solve differential equation set by common means, are to be avoided.

The effect and convenience of the grey systematic prediction has been illustrated by many instances; for example, to forecast grain yield, applying fertilizers, yield per mu and irrigated area together in Shanxi Province [79].

### Grey Decision-Making

Grey decision-making is primarily concerned with the grey strategy of situation, grey group decision making and grey programming.

### Grey Strategy of Situation

Grey strategy of situation deals with the strategy-making based on multi-objects which are contradictory in the ordinary way. The situation named by Grey System theory implies that the events to be attended to are coped over by games; then we have

$$s = (a, b) = (\text{event, game})$$

where  $s$  is of an appointed situation,  $a$  the event,  $b$  the game to be used to cope over the event.

The following are the essentials to make a grey strategy of the situation [22, 46]:

1. Exploiting point-set topology: the construction of situation set can be determined when an event is uncertain and games are unique. Let  $\mathbf{A}, \mathbf{B}$  be collections for events and games respectively, then we have

$$\mathbf{X} = \mathbf{A} \times \mathbf{B}, \quad a_j \in \mathbf{A}, \quad b_j \in \mathbf{B}$$

$$\varphi = \{s | s = \prod_{i=1}^2 Y_i \text{ when } i \neq j, \text{ then } Y_i = \mathbf{A}, Y_j = u_j \in \mathbf{O}_j, j = 1, 2\}$$

where  $\mathbf{O}$  is a system of open sets,  $\varphi$  is the sub-basis of  $x$ ,  $x$  is the topology of situation.

2. To discover the effects of a given game dealing with a raised event, let us consider the map  $\sigma$ .

$$\sigma: \{s_{ij}\} \xrightarrow{p} \{u_{ij}^{(p)}\}, s_{ij} \in \mathbf{X}$$

where  $\sigma$  is an effect map from situation  $s_{ij}$  to its sample  $u_{ij}^{(p)}$  describing the effect value concerning the  $p$ th object. By arranging the effect samples of all the objects,  $p = 1, 2, \dots, m$ , the following effect series is obtained

$$\mathbf{u}_{ij} = \{u_{ij}^{(1)}, u_{ij}^{(2)}, \dots, u_{ij}^{(m)}\}$$

3. Consider a grey relational space  $(\mathbf{X}, \Gamma)$  where  $\mathbf{X}$  is a collection of  $u_{ij}$ ,  $\Gamma$  is a map set in relational space. Assuming the grey target is

$$s_\varepsilon(\gamma) = \{\gamma(u_0, u_{ij}) | 1 - \gamma(u_0, u_{ij}) \geq \varepsilon\}$$

where  $u_0$  is a reference sample series, which implies the kernel of target, then  $s_{ij}$  is a satisfactory situation and  $b_j$  the satisfactory game dealing with event  $a_i$ , iff

$$\begin{aligned} u_{ij} &\in s_\varepsilon(\gamma) \\ u_{ij} &\Rightarrow s_{ij} \Rightarrow (a_i, b_j) \Rightarrow b_j \end{aligned}$$

It is also important to make a satisfactory strategy by means of effect measure maps, which transfer the disconformity samples resulting from different objects into identical scalars.

A grey variety of problems have been solved using the grey strategy of situation, such as the irrigation strategy in Henan Province [26], a forestation strategy in Jinagsu Province and the agricultural economic division in Shanxi Province [93, 118].

### Decision-Making of Group

A comprehensive treatment of the strategy having three levels, by means of grey relational space, grey statistics, grey clustering and grey prediction, is referred to as a decision-making of group [22, 28, 46].

Citing a case as a group strategy, let us consider an investment strategy whose items to be decided may be an exploited products—a category of investment, namely, the invested number and range which are to be marked by “much”, “middle” and “less”. The decision levels, namely, the collections of the decision and policy makers, are mass groups whose strategy is usually

determined solely by grey statistics; the expert group whose original strategy is obtained by grey prediction as usual and whose attention is paid only to the exploited products; the final group of the three levels, namely, the managers group, whose master strategy comes from official documentation and the quantitative strategy concerned is obtained by grey clustering in an ordinary way, taking only the category of investment into consideration.

A comprehensive strategy which gives consideration to the three levels can be obtained by unifying the decision series referred to the three groups, respectively, based on grey relational space.

A case in the above-mentioned point is one to harness a channel in Henan Province [26].

### Grey Programming

To confirm traditional programming as a foundation, to incorporate grey prediction model in order to make a dynamic programming and to regard the coefficients as grey numbers with provision for adapting to the environment, are the essential ideas and attempts of grey programming [28, 46].

Let us denote the model of linear programming by

$$\begin{aligned} f(x) &= \mathbf{c}^T x = \otimes \\ \mathbf{A}x &\leq \hat{b} \end{aligned}$$

where  $\otimes$  is a grey target. While the vector  $\mathbf{c}$  is referred to as a grey one whose elements are grey numbers denoted with  $\otimes(c_i)$ ,  $\mathbf{A}$  is a grey matrix denoted by

$$\begin{aligned} \mathbf{A} &= (\otimes_{ij}) \\ i &= 1, 2, \dots, n \\ j &= 1, 2, \dots, m \end{aligned}$$

$\hat{b}(\xi)$  represents a boundary vector at time  $\xi$  which is obtained with GM(1,1) modelling can be written as follows:

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO}: b_k \rightarrow \hat{b}_k(\xi)$$

where  $b_k$  is a time series associated with the  $k$ th coefficient belonging to boundary vector  $b$ , and  $\hat{b}_k(\xi)$  means the value of  $b_k$  to be forecasted at future time  $\xi$ , which constitutes  $\hat{b}(\xi)$ .

Then  $f(x^*)$  means a satisfactory cost and  $x^*$  is a satisfactory resolution of it, iff  $f(x^*)$  falls in a grey target which is given.

Grey linear programming has been brought to practical use, for example,

to adjust the structure of village economy [81] and to make a development plan coordinating the economy, society and science in Hubei Province.

## Grey Prediction Control

The control principle of conventional control theory, whether classical or modern, is to control the system behaviour according to the state sample which has already occurred. It is then referred to as an afterward manner for control. The demerits of the afterward manner are as follows:

1. Impossible to avert accidents in advance;
2. Impossible to control timely;
3. Weakly adaptable.

The essential idea of grey prediction control [13, 29, 49, 50] is to control the system behaviour in advance with the control strategy obtained from the prediction controller [6], based on GM(1,1). Figure 1 illustrates the outline for a grey prediction control system. It maintains a desired state within reasonably accurate tolerances even though the output  $y$  is varied.

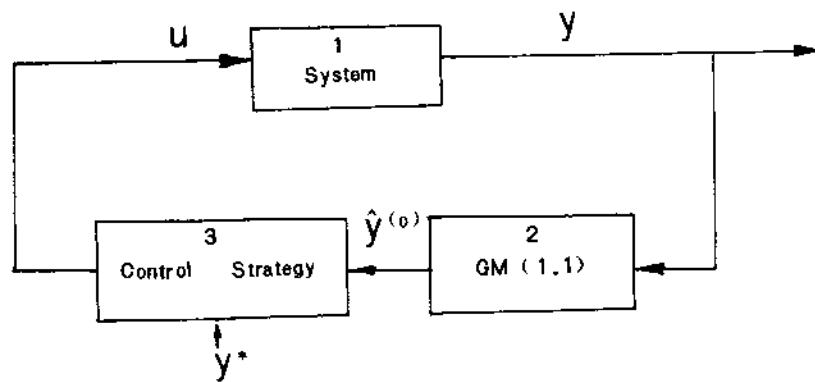


Fig. 1. Grey prediction control system.

In Fig. 1 the output of loop 1 (control object) is denoted as  $y$ , the prediction value of the systems behaviour is denoted as  $\hat{y}^{(0)}$  which is obtained from controller 2, the assigned quantity is denoted as  $y^*$  and  $u$  is a control strategy coming from loop 3.

The control principle is as follows: a transducer feeds the sampled data to prediction controller 2, which functions as a computer. The prediction value  $\hat{y}^{(0)}$  as a calculated result of controller 2 is delivered to loop 3, which constitutes a control strategy as compared with  $y^*$ . The control strategy  $u$  stems

from loop 3 as an input, activating loop 3, and then stored therein. Should the behaviour  $y^{(0)}$  occur in the future, the stored strategy  $u$  will deal with it and lead the systems behaviour to a desired state.

A practical grey prediction controller has been initiated by [6].

## References

- [1] Cao Yanying (1987). Modelling for grey forecasting of calamities in geography. *Youthgeographers*, 2, 6-11 (in Chinese).
- [2] Chen Mianyun (1982). Grey dynamic of the systems of a boring machine. *Journal of Huazhong University of Science and Technology*, 6, 7-11 (in Chinese).
- [3] Chen Mianyun (1985). The stabilization and stability of grey systems. *Fuzzy Mathematics, (Special Issue of Grey Systems)* 2, 101-6 (in Chinese).
- [4] Chen Mianyun (1988). Grey dynamic modelling and prediction control of macroeconomic systems. In: *Grey System*, China Ocean Press, Beijing, pp. 211-19.
- [5] Chen Mianyun (1988). Stability and regulation problems of grey systems. In: *Grey Systems*, China Ocean Press, Beijing, pp. 220-8.
- [6] Cheng Biao (1986). The grey control on industrial process. *Journal of Huangshi College*, 1, 11-23 (in Chinese).
- [7] Deng Julong (1982a). Control problems of Grey Systems. *Systems and Control Letters*, 5, 288-94.
- [8] Deng Julong (1982b). The least parameters controller of decetralized control system. *Acta Automatica Sinica*, 1, 49-54 (in Chinese).
- [9] Deng Julong (1982c). Grey systems control. *Journal of Huazhong University of Science and Technology*, 3, 9-18 (in Chinese).
- [10] Deng Julong (1983a). Grey System theories and measurement futurics. *Future and Development*, 3, 20-3 (in Chinese).
- [11] Deng Julong (1983b). Grey fuzzy forecast and control for grain. *Journal of Huazhong University of Science and Technology*, 2, 1-8 (in Chinese).
- [12] Deng Julong (1983c). Group of Grey Systems. *Journal of Huazhong University of Science and Technology*, 4, 15-24 (in Chinese).
- [13] Deng Julong (1983d). Survey of Grey System. *World Science*, 7, 1-5 (in Chinese).
- [14] Deng Julong (1984a). The theory and method of socioeconomic Grey Systems. *Social Sciences in China*, 6, 47-60 (in Chinese).
- [15] Deng Julong (1984b). The differential dynamic model (GM) and its implement in long period forecasting of grain. *Exploration of Nature*, 3, 47-50 (in Chinese).
- [16] Deng Julong (1984c). Grey nonlinear programming. *Exploration of Nature*, 3, 47-50 (in Chinese).
- [17] Deng Julong (1985a). Grey Systems. *Zhengzhou Hydrology*, 1, 1-10; 2, 2-9 (in Chinese).
- [18] Deng Julong (1985b). Relational space of Grey Systems. *Fuzzy Mathematics, (Special Issue of Grey Systems)* 2, 1-10 (in Chinese).
- [19] Deng Julong (1985c). Generated function of Grey Systems. *Fuzzy Mathematics, (Special Issue of Grey Systems)* 2, 11-22 (in Chinese).
- [20] Deng Julong (1985d). The GM model of Grey Systems. *Fuzzy Mathematics, (Special Issue of Grey Systems)* 2, 23-32 (in Chinese).
- [21] Deng Julong (1985e). Five kinds of Grey forecasting. *Fuzzy Mathematics, (Special Issue of Grey Systems)* 2, 33-42 (in Chinese).

- [22] Deng Julong (1985f). Grey state decision making. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 43–50 (in Chinese).
- [23] Deng Julong (1985g). Grey system and agriculture. *Agricultural Science in Shanxi*, 5, 34–7; 6, 29–33 (in Chinese).
- [24] Deng Julong (1985h). *Grey Systems: society, economy*, National Defence Press, Beijing, (in Chinese).
- [25] Deng Julong (1985i). *Grey Systems Control*, Huzhong University of Science and Technology Press, Wuhan, (in Chinese).
- [26] Deng Julong & Deng Qi (1985). On grey and fuzzy decision of lining building of irrigation channels. *System Science and Comprehensive Studies in Agriculture* 2, 26–30 (in Chinese).
- [27] Deng Julong & Zhou Chaoshun (1985). The feedback control and sufficient criteria for the stability of grey linear time-invariant systems. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 107–111 (in Chinese).
- [28] Deng Julong (1986a). *Grey Prediction and Grey Decision Making*, Huazhong University of Science and Technology Press, Wuhan, (in Chinese).
- [29] Deng Julong (1986b). Grey forecasting controller. *Exploration of Nature*, 4, 59–68 (in Chinese).
- [30] Deng Julong (1986c). The application of Grey System to quantitative investment strategy. *Investment and reform*, 3, 21–5 (in Chinese).
- [31] Deng Julong (1986d). Some essential approaches to latent Grey Systems. *Systems Engineering Theory and Practice*, 1, 66–72 (in Chinese).
- [32] Deng Julong (1986e). Essential problems for Grey System theory. *Journal of Xiangfan University*, 1, 5–27 (in Chinese).
- [33] Deng Julong (1986f). Grey Systems. *Journal of Huangshi College*, 1, 1–10 (in Chinese).
- [34] Deng Julong (1987a). *The essential methods of Grey Systems*, Huazhong University of Science and Technology Press, Wuhan, (in Chinese).
- [35] Deng Julong (1987b). Properties of grey forecasting model GM(1,1). *Journal of Huazhong University of Science and Technology*, 5, 1–6 (in Chinese).
- [36] Deng Julong (1987c). Grey exponential law for accumulative generation. *Journal of Huazhong University of Science and Technology*, 5, 7–12 (in Chinese).
- [37] Deng Julong (1987d). Grey decision making and its use for the determination of irrigation strategy. In: *Optimization Models Using Fuzzy Sets and Possibility Theory*, eds J. Kacprzyk & S.A. Orlovski. D. Reidel Publishing Company, Netherlands pp. 447–58.
- [38] Deng Julong & Zhou Chaoshun (1987). Sufficient condition for stability of Grey Systems and its application in the analysis of stability of linear time-varying systems. *Information and Control*, 4, 24–7 (in Chinese).
- [39] Deng Julong (1988a). Properties of relational space for Grey Systems. In: *Grey System*, China Ocean Press, Beijing, pp. 1–13.
- [40] Deng Julong (1988b). Properties of generating function for Grey Systems. In: *Grey System*, China Ocean Press, Beijing, pp. 4–30.
- [41] Deng Julong (1988c). The grey exponential law of AGO. In: *Grey System*. China Ocean Press, Beijing, pp. 13–39.
- [42] Deng Julong (1988d). Modelling of the GM model of Grey Systems. In: *Grey System*, China Ocean Press, Beijing, pp. 40–53.
- [43] Deng Julong (1988e). Grey forecasting model. In: *Grey System*, China Ocean Press, Beijing, pp. 54–69.
- [44] Deng Julong (1988f). Properties of the grey forecasting model of GM(1,1). In: *Grey System*, China Ocean Press, Beijing, pp. 79–90.
- [45] Deng Julong (1988h). Five step modelling (FSM) of Grey Systems In: *Grey System*,

- China Ocean Press, Beijing, pp. 91–102.
- [46] Deng Julong (1988i). Grey strategy of situation, In: *Grey System*, China Ocean Press, Beijing, pp. 103–14.
- [47] Deng Julong (1988j). Grey decision making of group. In: *Grey System*, China Ocean Press, Beijing, pp. 115–29.
- [48] Deng Julong (1988k). Grey linear programming. In: *Grey System*, China Ocean Press, Beijing, pp. 130–8.
- [49] Deng Julong (1988l). Grey forecasting control. In: *Grey System*, China Ocean Press, Beijing, pp. 138–49.
- [50] Deng Julong (1988m). Process adaptation of GM(1,1). In: *Grey System*, China Ocean Press, Beijing, pp. 150–64.
- [51] Deng Qi, Feng Qingxi & Deng Julong (1985). Underground water level control system and optimal grey decision making (irrigation decision making for the People's Victory Channel). *Acta Automatica Sinica*, 2, 131–6 (in Chinese).
- [52] Gong Guoqing (1987). Calculating dropping water level with grey model. *Journal of CESU Youth Scientist Association*, 1, 81–5 (in Chinese).
- [53] Guo Hong (1985). Identification coefficient of relational grade of Grey Systems. *Fuzzy Mathematics*, 2, 55–8 (in Chinese).
- [54] Guo Hong (1986a). Controlling for Grey System. *Journal of Xiangfan University*, 1, 28–32 (in Chinese).
- [55] Guo Hong (1986b). Grey classification of medical diagnosis. *Exploration of Nature*, 4, 69–75 (in Chinese).
- [56] Guo Hong (1986c). Grey System theory. *Interdisciplinary Science*, 1, 17–8 (in Chinese).
- [57] Guo Hong (1988). Extension of the norm interval for relational space of Grey System theory. In: *Grey System*, China Ocean Press, Beijing, pp. 165–70.
- [58] Hu Congwu (1987). The trend of yield with drought or excessive rain in Guangshan Country. *System Sciences and Comprehensive Studies in Agriculture*, 1, 39–42 (in Chinese).
- [59] Hu Tao & Huang Xiaoqing (1985). Analysis of factors harming the Korean Pine—applying Grey System theory to the reasearch on ecosystem. *Fuzzy Mathematics*, 2, 51–4 (in Chinese).
- [60] Hu Xingdou (1987). Decomposition analysis and stability of Grey Systems. *Journal of Beijing Institute of Technology*, 5, 45–50 (in Chinese).
- [61] Huang Jianer (1987). Applications of grey statistics to investment decision making of forestry. *Economic Problems of Forestry*, 2, 44–51 (in Chinese).
- [62] Huang jimin (1985). Building of grey forecasting model for grain yield prediction in Jiangxi Province. *Journal of Jiangxi Normal University*, 3, 47–52 (in Chinese).
- [63] Huo Junjiang (1986a). On the applications of Grey System Theory and methods to history studies. *Social Science* 10, 64–8 (in Chinese).
- [64] Huo Junjiang (1986b). Grey System methods and investigation. *Seeker*, 4, 94–8 (in Chinese).
- [65] Jiang Ye et al. (1987). Applying grey group to the devision of the suitable climate ecocondition Panax Oinguefolium grown. *System Sciences and Comprehensive Studies in Agriculture*, 4, 64–70 (in Chinese).
- [66] Jiao Jiujiu (1987). Adjustment and control for hydrogeology system's dynamic. *Journal of CESU Youth Scientist Association*, 1, 74–81 (in Chinese).
- [67] Lec Senra (1986). Grey forecasting the freight volume for vehicle of railway. *Journal of Xiangfan University*, 1, 33–5 (in Chinese).
- [68] Li Biaolin (1987). Models of grey mutation in ecoagriculture. *System Sciences and Comprehensive Studies in Agriculture*, 3, 39–47 (in Chinese).
- [69] Li Biaolin & Du Deshou (1984). Preliminary investigation of application of Grey Sys-

- tem in ecological engineering the impact of Ladybird Beetle on Cotton Aphid. *Journal of Huazhong University of Science and Technology*, 3, 1-8 (in Chinese).
- [70] Li Biaolin & Deng Julong (1984). The grey model of biological prevention and cure systems of Aphis gossypii Glover. *Exploration of Nature*, 3, 44-6 (in Chinese).
- [71] Li Xueping & Chen Shaohua (1987). Grey adaptation for linear system and adaptive control by grey forecast. *Academic Research*, 4, 58-64 (in Chinese).
- [72] Li Xueping & Liu Huizhong (1987). On scientific methods in Grey System. *Exploration of Nature*, 4, 125-30 (in Chinese).
- [73] Li Zhongmo (1986). Primary application of the Grey System theory in the study of earthquake forecast. *Journal of Seismology*, 4, 27-31 (in Chinese).
- [74] Li Yaoxian & Zhao Yufu (1987). The application of Grey System theory to long-range prediction. *Acta Meteorologica Sinica*, 4, 489-94. (in Chinese).
- [75] Liu Guangji & Luo Gaorong (1986). Grey System predicting theory applied to establishing equation for forecasting annual consumption of electrical power. *Systems Engineering Theory and practice*, 3, 38-41 (in Chinese).
- [76] Long Yu (1987). Identifying system's factors with grey relational method. *Journal of Xiangfan University*, 1, 51-5 (in Chinese).
- [77] Luo Jianjun (1987). Quantitative analysis of agricultural advancing in Shanxi Province. *System Sciences and Comprehensive Studies in Agriculture*, 1, 43-50 (in Chinese).
- [78] Luo Jianjun, Wang Xuemeng & Nie Hongsheng (1985). Some instances for applications of Grey System theory to agriculture in Shanxi Province. *Agricultural Science in Shanxi*, 10, 24-6 (in Chinese).
- [79] Luo Jianjun & Zhang Bingxiang (1986). An analysis on the grey forecast and the control of grain production in Shanxi Province. *Acta Agricultural Boreali Sinica*, 1, 66-71 (in Chinese).
- [80] Luo Qingcheng (1985). Tentative exploration of the Grey System theory to the agricultural input-output simulation. *System Science and Comprehensive Studies in Agriculture*, 3, 24-30 (in Chinese).
- [81] Luo Qingcheng (1987). Grey linear programming and its appliance for agricultural structure optimization. *System Sciences and Comprehensive Studies in Agriculture* 10, 49-55 (in Chinese).
- [82] Ma Xianyao & Zhen Ping (1985). The forecasting of creep behaviour for low alloy steel. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 85-8 (in Chinese).
- [83] Ma Xianyao & Zhen Ping (1986). Research on impact strength of ferritic apheroidal graphite cast iron by grey system theory. *Fuzzy Mathematics*, (Special Issue of Grey System) 2, 89-98 (in Chinese).
- [84] Ma Zhiying & Shi Shuyan (1988). Application of grey relational grade analysis in cotton lines evaluation. *Journal of Hebei Agricultural University*, 4, 121-8 (in Chinese).
- [85] Meng Wanrong (1985). Application of Grey System theory to forecasting technological progress in industry owned by the whole people in Hubei Province. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 97-100 (in Chinese).
- [86] Meng Wanrong (1988). Application of the Grey System theory to forecasting the technical progress in the industries of Hubei Province. In: *Grey System*, China Ocean Press, Beijing, pp. 175-9.
- [87] Tan Chengye (1986). Correlation degree and fuzzy cluster analysis on strong earthquake precursors. *Journal of Seismological Research*, 4, 409-17 (in Chinese).
- [88] Tan Chengye, Ruan Shengna & Yang Guifeng (1986). Discussions on the establishment of the seismological grey model. *Journal of Seismological Research*, 6, 685-92 (in Chinese).
- [89] Wang Aicheng & Yu Min (1988). Applications of grey forecasting method to diseases prediction. *Journal of Epidemiology of China*, 1, 49-52 (in Chinese).

- [90] Wang Min & Jiao Jiujiu (1987). Application studies of hydrogeology with Grey System theory. *Journal of CESU Youth Scientist Association*, 1, 65-74 (in Chinese).
- [91] Wang Mingxi & Liu Wan (1985). Long period forecasting of first frost by Grey System theory. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 59-66 (in Chinese).
- [92] Wang Qingyin (1987). Situation analysis of the development of machine building industry in the City of Handan. *Systems Engineering—Theory and Practice*, 2, 43-6 (in Chinese).
- [93] Wang Shumin (1985). The grey state strategy of multitarget for agricultural division. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 67-76 (in Chinese).
- [94] Wang Ting (1985). Grey functions and relational grade. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 77-80 (in Chinese).
- [95] Wang Xuemeng & Luo Jianjun (1986). Grey dynamic model and its analysis of the circle. *Systems Sciences and Comprehensive Studies in Agriculture*, 1, 23-6 (in Chinese).
- [96] Wang Zhixue (1988). The grey calculating analysis of Grand Jujube output in Fuping County. *Journal of Hebei Forestry College*, 1, 24-32 (in Chinese).
- [97] Wei Zhongzhe & Yang Zheba (1986). The macro control model for the teachers of university. *Applications of System Engineering*, 1, 32-9 (in Chinese).
- [98] Xia Jun (1987). Estimating influences by the environment for exploiting the water sources. *Social Science Review of Journal of Wuhan Hydrology and Power Institute*, 1, 97-104 (in Chinese).
- [99] Xin Jianjun et al. (1987). Applying the Grey System theory to the evaluation land quality. *System Sciences and Comprehensive Studies in Agriculture*, 4, 39-45 (in Chinese).
- [100] Xiong Jiling (1988). Analysis of potentials of the load of judicial system of China. In: *Grey System*, China Ocean Press, Beijing, pp. 195-210.
- [101] Xu Dianxiong (1986). Analysis of GM (1,1) model assembly language algorithm. *Journal of Huangshi College*, 1, 25-31 (in Chinese).
- [102] Xu Shaoyan (1988). Analysis of GM (1,N) forecasting model and its applications. In: *Grey System*, China Ocean Press, Beijing, pp. 180-94.
- [103] Yan Wenbing & Chang Lianfang (1986). Forecasting and analysis for the average population incomes of farmers of Shanxi Province in 1986. *Prediction of Expenditure Tendency*, 6, 42-4 (in Chinese).
- [104] Yan Wenbing & Luo Jianjun (1986). An analysis of dominant economy and a selection of the most economic model on the region. *System Sciences and Comprehensive Studies in Agriculture*, 1, 60-6 (in Chinese).
- [105] Yi Desheng (1987). Grey model (GM) and technician forecast. *Exploration of Nature*, 4, 75-81 (in Chinese).
- [106] Yi Desheng (1987). Grey series forecasting. *Forecasting*, 4, 14-7 (in Chinese).
- [107] Yao Jianmin (1987). The problem for determining the accuracy of the grey forecast value. *System Sciences and Comprehensive Studies in Agriculture*, 3, 18-22 (in Chinese).
- [108] Yu Liyun (1986). The grey forecast for urban domestic wastes. *Environment Pollution and Control*, 5, 7-9 (in Chinese).
- [109] Yuan Jiazu (1988). The grey prediction model of the Populus sp growth increment. *Journal of Hebei Forestry College*, 1, 15-23 (in Chinese).
- [110] Yang Yunhua & Wang Ting (1985). Measurement and operation of grey concept. *Fuzzy Mathematics*, (Special Issue of Grey Systems) 2, 81-4 (in Chinese).
- [111] Zeng Guoning (1987). Applications of Grey System theory and methods to estimate the business benefits. *Development Studies*, 4, 54-6 (in Chinese).
- [112] Zhan Yihui (1982). The synthetic controllability of Grey System. *Journal of*

- Huazhong University of Science and Technology*, 6, 11–6 (in Chinese).
- [113] Zhang Qifan (1982). A preliminary study of the whitening of the Grey System. *Journal of Huazhong University of Science and Technology*, 3, 19–22 (in Chinese).
- [114] Zhang Bingxiang & Luo Jianjun (1985). Grey dynamic model (GM) with application to grain prediction. *System Engineering*, 4, 36–7 (in Chinese).
- [115] Zhang Huimin (1986). A grey forecast of tendency to dry from 1985 to 2000 in Luoyang Area. *System Sciences and Comprehensive Studies in Agriculture*, 1, 36–40 (in Chinese).
- [116] Zhang Dianqing (1986). Grey relational analysis of the shape function and the level of body quality for youngsters and children. *Sport Science in Guizhou Province*, 2, 1–5 (in Chinese).
- [117] Zhang Guangzheng et al. (1987). System analysis and forecast of farmer's income and its construction with Grey System theory. *System Sciences and Comprehensive Studies in Agriculture*, 1, 57–70 (in Chinese).
- [118] Zhang Jizhong et al. (1987). Quantitative research in classification of countryside economy in Shanxi Province. *System Sciences and Comprehensive Studies in Agriculture*, 1, 17–23 (in Chinese).
- [119] Zhao Songnian (1987). On decision of macrosystems. *Exploration of Nature*, 4, 119–28 (in Chinese).
- [120] Zhao Yunsheng (1987a). Grey modelling for the parameters of ventilating duct. *Journal of CESU Youth Scientist Association*, 1, 55–65 (in Chinese).
- [121] Zhao Yunsheng (1987b). Theory of establishment of grey models and its uses in calculation of ventilation variables. *Nonferrous Metals*, 4, 18–25 (in Chinese).
- [122] Zhou Ru (1986). The estimation of satisfactory soil constitution by means of grey clustering in North of the Huihe River. *Application of System Engineering*, 1, 40–52 (in Chinese).

## Properties of Multivariable Grey Model GM(1,N)\*

Deng Julong

Department of Automation, Huazhong University of Science and Technology, Wuhan, People's Republic of China

Received October 1988

**Abstract**—This paper develops an exponential solution of the GM(1,N) and studies of its some properties.

**Keywords:** grey modelling, GM, multivariable grey model.

### Introduction

A multivariable grey model denoted by the GM(1,N), which, developed by contributions [1, 2, 3, 4], includes  $N$  variables has been applied to a variety of fields for the purpose of modelling and analysing multifactor affairs and problems. But how to obtain conveniently a solution and how to improve the modelling accuracy to yield an appropriate result are still important objectives. Now, let us find an exponential solution for the GM(1,N).

### Exponential Model

**Definition 1.** Let  $x(t)$  be a continuous function,  $\Delta t$  the time gap between  $x(t)$  and  $x(t+\Delta t)$ , and  $d(x,\Delta t)$  a derivative of  $x(t)$  which satisfies the following condition

$$d(x,\Delta t) = \lim_{\Delta t \rightarrow 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

where  $d(x,\Delta t) = \frac{dx}{dt}$

\* The project was supported by the National Science Foundation of China.